

Atividade de diagnóstico

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1.1. $\sin \alpha = \frac{b}{a}$, $\cos \alpha = \frac{c}{a}$ e $\tan \alpha = \frac{b}{c}$

1.2. a) $\sin \alpha = \frac{30}{50} = \frac{3}{5}$; $\cos \alpha = \frac{40}{50} = \frac{4}{5}$ e $\tan \alpha = \frac{30}{40} = \frac{3}{4}$

b) $\sin \alpha = \frac{4,5}{7,5} = \frac{3}{5}$; $\cos \alpha = \frac{6}{7,5} = \frac{4}{5}$ e $\tan \alpha = \frac{4,5}{6} = \frac{3}{4}$

2.1. $\sin 25^\circ = \cos(90^\circ - 25^\circ) = \cos 65^\circ$

2.2. $\cos(a - 30^\circ) = \sin[90^\circ - (a - 30^\circ)] = \sin(90^\circ - a + 30^\circ) = \sin(120^\circ - a)$

3. $\sin^2 \alpha + \cos^2 \alpha = 1$

$$\sin^2 \alpha + \left(\frac{2}{3}\right)^2 = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{4}{9} \Leftrightarrow \sin^2 \alpha = \frac{5}{9}$$

Como $\sin \alpha > 0$, $\sin \alpha = \frac{\sqrt{5}}{3}$.

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$$

4.1. $\tan^2 \alpha = 25 \Leftrightarrow \frac{\sin^2 \alpha}{\cos^2 \alpha} = 25 \Leftrightarrow \sin^2 \alpha = 25 \cos^2 \alpha$

$$\begin{aligned} \sin^2 \alpha + \cos^2 \alpha &= 1 \Leftrightarrow \\ &\Leftrightarrow 25 \cos^2 \alpha + \cos^2 \alpha = 1 \Leftrightarrow \\ &\Leftrightarrow 26 \cos^2 \alpha = 1 \Leftrightarrow \\ &\Leftrightarrow \cos^2 \alpha = \frac{1}{26} \end{aligned}$$

Como $\cos \alpha > 0$, então $\cos \alpha = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$.

4.2. $\sin^2 \alpha = 25 \cos^2 \alpha$

$$\sin^2 \alpha = 25 \times \frac{1}{26} = \frac{25}{26}$$

Como $\sin \alpha > 0$, então $\sin \alpha = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$.

4.3. $2 \sin \alpha \times \cos \alpha = 2 \times \frac{5\sqrt{26}}{26} \times \frac{\sqrt{26}}{26} = \frac{2 \times 5 \times 26}{26 \times 26} = \frac{10}{26} = \frac{5}{13}$

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5.1. $\sin 60^\circ - 2 \tan 45^\circ + \cos^2 45^\circ =$

$$\begin{aligned} &= \frac{\sqrt{3}}{2} - 2 \times 1 + \left(\frac{\sqrt{2}}{2}\right)^2 = \\ &= \frac{\sqrt{3}}{2} - 2 + \frac{1}{2} = \frac{\sqrt{3}}{2} - \frac{3}{2} = \frac{\sqrt{3} - 3}{2} \end{aligned}$$

5.2. $\sin^2 45^\circ + \cos^2 60^\circ + \tan^3 45^\circ =$

$$\begin{aligned} &= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1^3 = \\ &= \frac{1}{2} + \frac{1}{4} + 1 = \frac{7}{4} \end{aligned}$$

6. $h^2 + \left(\frac{1}{2}\right)^2 = 1^2 \Leftrightarrow h^2 = 1 - \frac{1}{4} \Leftrightarrow h^2 = \frac{3}{4}$

Logo, $h = \frac{\sqrt{3}}{2}$.

6.1. $\sin 30^\circ = \frac{AM}{AC} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

6.2. $\tan 30^\circ = \frac{AM}{MC} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2\sqrt{3}} = \frac{\sqrt{3}}{3}$

7. $\overline{BC}^2 = 2,3^2 + 5,2^2 \Leftrightarrow \overline{BC} = \sqrt{32,33}$

$$\tan \alpha = \frac{2,3}{5,2} \approx 0,4; \sin \alpha = \frac{2,3}{\sqrt{32,33}} \approx 0,4 \text{ e}$$

$$\cos \alpha = \frac{5,2}{\sqrt{32,33}} \approx 0,9$$

8. $\alpha \approx \tan^{-1}(2) \approx 63,4^\circ$

9.1. $\overline{BC}^2 = 4^2 + 3^2$
 $\overline{BC} = \sqrt{16+9} \Leftrightarrow \overline{BC} = 5$
 $\overline{BC} = 5 \text{ cm}$

9.2. a) $\tan(\hat{CBA}) = \frac{3}{4}$

Logo, $\hat{CBA} = \tan^{-1}\left(\frac{3}{4}\right) \approx 36,9^\circ$.

b) $\hat{ACB} = 90^\circ - \hat{CBA} \approx 90^\circ - 36,87^\circ \approx 53,1^\circ$

10.1. $\hat{ACB} = 90^\circ - 20^\circ = 70^\circ$

10.2. $\frac{\overline{AC}}{\overline{AB}} = \tan 20^\circ \Leftrightarrow \frac{\overline{AC}}{6} = \tan 20^\circ \Leftrightarrow \overline{AC} = 6 \times \tan 20^\circ$
 $\overline{AC} \approx 2,2 \text{ cm}$
 $\frac{\overline{AB}}{\overline{BC}} = \cos 20^\circ \Leftrightarrow 6 = \overline{BC} \times \cos 20^\circ \Leftrightarrow \overline{BC} = \frac{6}{\cos 20^\circ}$
 $\overline{BC} \approx 6,4 \text{ cm}$

Atividade inicial 1

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$$\frac{h}{81} = \sin 82^\circ \Leftrightarrow h = 81 \sin 82^\circ$$

$$\frac{h}{a} = \sin 58^\circ \Leftrightarrow h = a \sin 58^\circ$$

$$a \sin 58^\circ = 81 \sin 82^\circ \Leftrightarrow$$

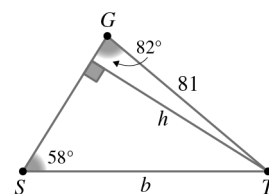
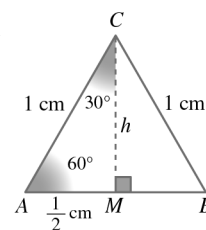
$$\Leftrightarrow a = \frac{81 \sin 82^\circ}{\sin 58^\circ} \Rightarrow a \approx 94,6$$

A distância a percorrer entre S. Jorge e a Ilha Terceira é de 94,6 km, aproximadamente.

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1.1. $A = 180^\circ - 60^\circ - 45^\circ = 75^\circ$

$$\frac{\sin 75^\circ}{4} = \frac{\sin 45^\circ}{b} = \frac{\sin 60^\circ}{c}$$



$$b = \frac{4 \times \sin 45^\circ}{\sin 75^\circ} = \frac{4 \times \frac{\sqrt{2}}{2}}{\sin 75^\circ} = \frac{2\sqrt{2}}{\sin 75^\circ} \approx 2,928$$

$$\overline{AC} \approx 2,9 \text{ cm}$$

$$c = \frac{4 \times \sin 60^\circ}{\sin 75^\circ} = \frac{4 \times \frac{\sqrt{3}}{2}}{\sin 75^\circ} = \frac{2\sqrt{3}}{\sin 75^\circ} \approx 3,586$$

$$\overline{AB} \approx 3,6 \text{ cm}$$

1.2. $B = 180^\circ - 63^\circ - 82^\circ = 35^\circ$

$$\frac{\sin 82^\circ}{16} = \frac{\sin 63^\circ}{a} = \frac{\sin 35^\circ}{b}$$

$$a = \frac{16 \sin 35^\circ}{\sin 82^\circ} \approx 14,396$$

$$\overline{BC} \approx 14,4 \text{ cm}$$

$$b = \frac{16 \sin 35^\circ}{\sin 82^\circ} \approx 9,267$$

$$\overline{AC} \approx 9,3 \text{ cm}$$

1.3. $A = 180^\circ - 85^\circ - 15^\circ = 80^\circ$

$$\frac{\sin 85^\circ}{20} = \frac{\sin 80^\circ}{a} = \frac{\sin 15^\circ}{c}$$

$$a = \frac{20 \times \sin 80^\circ}{\sin 85^\circ} \approx 19,771$$

$$\overline{BC} \approx 19,8 \text{ cm}$$

$$c = \frac{20 \times \sin 15^\circ}{\sin 85^\circ} \approx 5,196$$

$$\overline{AB} \approx 5,2 \text{ cm}$$

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2. $\sin 150^\circ + \sin 30^\circ + \sin 90^\circ = \sin(180^\circ - 30^\circ) + \frac{1}{2} + 1 =$

$$= \sin 30^\circ + \frac{3}{2} = \frac{1}{2} + \frac{3}{2} = 2$$

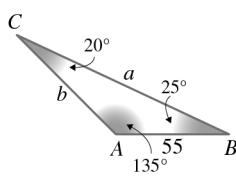
3. $180^\circ - 20^\circ - 25^\circ = 135^\circ$

$$\frac{\sin 20^\circ}{55} = \frac{\sin 135^\circ}{a} = \frac{\sin 25^\circ}{b}$$

$$a = \frac{55 \times \sin 135^\circ}{\sin 20^\circ} \approx 113,7$$

$$b = \frac{55 \times \sin 25^\circ}{\sin 20^\circ} \approx 68,0$$

Logo, $\overline{AC} \approx 68,0 \text{ cm}$ e $\overline{BC} \approx 113,7 \text{ cm}$.



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4.1. $a^2 = b^2 + c^2 - 2bc \cos A$

$$a^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \cos 26^\circ \Leftrightarrow$$

$$\Leftrightarrow a^2 = 34 - 30 \cos 26^\circ \Rightarrow$$

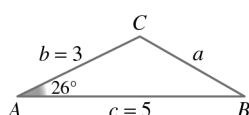
$$\Rightarrow a^2 \approx 7,0362$$

$$\overline{BC} = a \approx \sqrt{7,0362} \approx 2,7$$

$$\overline{BC} \approx 2,7 \text{ cm}$$

4.2. $\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{2,7^2 + 5^2 - 3^2}{2 \times 2,7 \times 5} \approx 0,8626$

$$B \approx \cos^{-1}(0,8626) \approx 30,4^\circ$$



5.1. $a = 4,8$; $b = 6,5$ e $c = 8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6,5^2 + 8^2 - 4,8^2}{2 \times 6,5 \times 8} = \frac{83,21}{104}$$

$$A = \cos^{-1}\left(\frac{83,21}{104}\right) \approx 36,86^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{4,8^2 + 8^2 - 6,5^2}{2 \times 4,8 \times 8} = \frac{44,79}{76,8}$$

$$B = \cos^{-1}\left(\frac{44,79}{76,8}\right) \approx 54,32^\circ$$

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4,8^2 + 6,5^2 - 8^2}{2 \times 4,8 \times 6,5} = \frac{1,29}{62,4}$$

$$c = \cos^{-1}\left(\frac{1,29}{62,4}\right) \approx 88,82^\circ$$

$$A \approx 36,86^\circ, B \approx 54,32^\circ \text{ e } C \approx 88,82^\circ$$

5.2. $a = 3$, $b = c = 5$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 5^2 - 3^2}{2 \times 5 \times 5} = \frac{41}{50}$$

$$A \approx \cos^{-1}\left(\frac{41}{50}\right) \approx 34,92^\circ$$

Como $b = c$, temos $B = C$.

$$B = C = \frac{180^\circ - 34,92^\circ}{2} = 72,54^\circ$$

$$A \approx 34,92^\circ, B \approx 72,54^\circ \text{ e } C \approx 72,54^\circ$$

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6. $A = 101^\circ$, $a = \overline{BC}$, $b = 7 \text{ cm}$, $c = 12 \text{ cm}$

6.1. $a^2 = b^2 + c^2 - 2bc \cos A = 7^2 + 12^2 - 2 \times 7 \times 12 \times \cos 101^\circ \approx$

$$\approx 225,056$$

$$a \approx \sqrt{225,056} \approx 15,0$$

$$\overline{BC} \approx 15,0 \text{ cm}$$

6.2. $\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{225,0559 + 12^2 - 7^2}{2 \times \sqrt{225,0559} \times 12} \approx 0,8889$

$$B \approx \cos^{-1}(0,8889) \approx 27,3^\circ$$

$$\hat{CBA} \approx 27,3^\circ$$

7. $\cos 120^\circ - \sin 150^\circ + \cos 90^\circ =$

$$= \cos(180^\circ - 60^\circ) - \sin(180^\circ - 30^\circ) + 0 =$$

$$= -\cos 60^\circ - \sin 30^\circ = -\frac{1}{2} - \frac{1}{2} = -1$$

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8.1. $a = 7 \text{ cm}$, $b = 8 \text{ cm}$ e $c = 9 \text{ cm}$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 9^2 - 7^2}{2 \times 8 \times 9} = \frac{96}{144} = \frac{2}{3}$$

$$A = \cos^{-1}\left(\frac{2}{3}\right) \approx 48,2^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{7^2 + 9^2 - 8^2}{2 \times 7 \times 9} = \frac{66}{126} = \frac{11}{21}$$

$$B = \cos^{-1}\left(\frac{11}{21}\right) \approx 58,4^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{32}{112} = \frac{2}{7}$$

$$C = \cos^{-1}\left(\frac{2}{7}\right) \approx 73,4^\circ$$

$$A \approx 48,2^\circ; B \approx 58,4^\circ \text{ e } C \approx 73,4^\circ$$

8.2. $a = 5 \text{ cm}, b = 8 \text{ cm e } c = 5 \text{ cm}$

Como $a = c$, temos $A = C$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 5^2 - 5^2}{2 \times 8 \times 5} = \frac{64}{80} = \frac{4}{5}$$

$$A = \cos^{-1}\left(\frac{4}{5}\right) \approx 36,9^\circ$$

$$C \approx 36,9^\circ$$

$$B \approx 180^\circ - 2 \times 36,9^\circ = 106,2^\circ$$

$$A \approx 36,9^\circ; B \approx 106,2^\circ \text{ e } C \approx 36,9^\circ$$

8.3. $a = 2,8 \text{ cm}; b \approx 4,5 \text{ cm e } c \approx 5,3 \text{ cm}$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4,5^2 + 5,3^2 - 2,8^2}{2 \times 4,5 \times 5,3} = \frac{40,5}{47,7}$$

$$A = \cos^{-1}\left(\frac{40,5}{47,7}\right) \approx 31,9^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2,8^2 + 5,3^2 - 4,5^2}{2 \times 2,8 \times 5,3} = \frac{15,68}{29,68}$$

$$B = \cos^{-1}\left(\frac{15,68}{29,68}\right) \approx 58,1^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2,8^2 + 4,5^2 - 5,3^2}{2 \times 2,8 \times 4,5} = 0$$

$$C = \cos^{-1}(0) = 90^\circ$$

$$A \approx 31,9^\circ; B \approx 58,1^\circ \text{ e } C = 90^\circ$$

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9.1. $A = 35^\circ, b = 6 \text{ cm e } c = 7 \text{ cm}$

$$a^2 = b^2 + c^2 - 2bc \cos A =$$

$$= 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 35^\circ \approx$$

$$\approx 16,1912$$

$$a \approx \sqrt{16,1912} \approx 4,0$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{16,1912 + 7^2 - 6^2}{2 \times \sqrt{16,1912} \times 7} \approx 0,5182$$

$$B = \cos^{-1}(0,5182) \approx 58,8^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16,1912 + 6^2 - 7^2}{2 \times \sqrt{16,1912} \times 6} \approx 0,0661$$

$$C \approx \cos^{-1}(0,0661) \approx 86,2^\circ$$

$$\overline{BC} \approx 4,0 \text{ cm}; B \approx 58,8^\circ \text{ e } C \approx 86,2^\circ$$

9.2. $B = 15^\circ, a = 5 \text{ cm e } c = 8 \text{ cm}$

$$b^2 = a^2 + c^2 - 2ac \cos B =$$

$$= 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 15^\circ \approx 11,7259$$

$$b \approx \sqrt{11,7259} \approx 3,424$$

$$\overline{AC} \approx 3,4 \text{ cm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{11,7259 + 8^2 - 5^2}{2 \times \sqrt{11,7259} \times 8} \approx 0,9258$$

$$A \approx \cos^{-1}(0,9258) \approx 22,2^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5^2 + 11,7259 - 8^2}{2 \times 5 \times \sqrt{11,7259}} \approx -0,7965$$

$$C \approx \cos^{-1}(-0,7965) \approx 142,8^\circ$$

$$\overline{AC} \approx 3,4 \text{ cm}; A \approx 22,2^\circ \text{ e } C \approx 142,8^\circ$$

9.3. $A = 40^\circ \text{ e } a = b = 5 \text{ cm}$

O triângulo $[ABC]$ é isósceles.

$$B = A = 40^\circ$$

$$C = 180^\circ - 2 \times 40^\circ = 100^\circ$$

$$c^2 = a^2 + b^2 - 2ab \cos C =$$

$$= 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 100^\circ \approx 58,6824$$

$$c \approx \sqrt{58,6824} \approx 7,660$$

$$\overline{AB} \approx 7,7 \text{ cm}$$

$$\overline{AB} \approx 7,7 \text{ cm}, B = 40^\circ \text{ e } C = 100^\circ$$

10. $A = 30^\circ, b = 6\sqrt{3} \text{ e } c = 9$

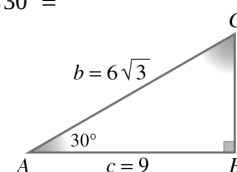
$$a^2 = b^2 + c^2 - 2bc \cos A =$$

$$= (6\sqrt{3})^2 + 9^2 - 2 \times 6\sqrt{3} \times 9 \times \cos 30^\circ =$$

$$= 108 + 81 - 2 \times 54\sqrt{3} \times \frac{\sqrt{3}}{2} =$$

$$= 27$$

$$a = \sqrt{27} = 3\sqrt{3}$$



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(3\sqrt{3})^2 + 9^2 - (6\sqrt{3})^2}{2 \times \sqrt{27} \times 9} = \frac{27 + 81 - 108}{18\sqrt{27}} = 0$$

$B = \cos^{-1}(0) = 90^\circ$ (O triângulo $[ABC]$ é retângulo em B.)

$$C = 90^\circ - 30^\circ = 60^\circ$$

$$\overline{BC} = 3\sqrt{3} \text{ cm}, B = 90^\circ \text{ e } C = 60^\circ$$

11. $a^2 = b^2 + c^2 - 2bc \cos A =$

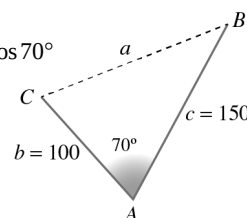
$$= 100^2 + 150^2 - 2 \times 100 \times 150 \times \cos 70^\circ$$

$$\approx 22\,239,396$$

$$a \approx \sqrt{22\,239,396} \approx 149,1$$

A distância entre os dois balões

é de 149 m, aproximadamente.



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12.1. $A = 72^\circ; C = 48^\circ \text{ e } c = 50 \text{ cm}$

$$B = 180^\circ - 72^\circ - 48^\circ = 60^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 72^\circ}{a} = \frac{\sin 60^\circ}{b} = \frac{\sin 48^\circ}{50}$$

$$a = \frac{50 \times \sin 72^\circ}{\sin 48^\circ} \approx 63,989$$

$$b = \frac{50 \sin 60^\circ}{\sin 48^\circ} \approx 58,268$$

$$P_{[ABC]} \approx (63,989 + 58,268 + 50) \text{ cm} \approx 172,3 \text{ cm}$$

12.2. $A = 25^\circ, C = 130^\circ \text{ e } b = 12 \text{ cm}$

$$B = 180^\circ - 25^\circ - 130^\circ = 25^\circ$$

O triângulo $[ABC]$ é isósceles: $\overline{BC} = \overline{AC} = 12 \text{ cm}$

$$a = 12 \text{ cm}$$

$$\frac{\sin 130^\circ}{c} = \frac{\sin 25^\circ}{12} \Leftrightarrow c = \frac{12 \times \sin 130^\circ}{\sin 25^\circ} \approx 21,751$$

$$P_{[ABC]} \approx (12 + 12 + 21,751) \approx 45,8 \text{ cm}$$

13. $\widehat{CBF} = 180^\circ - 90^\circ - 55^\circ = 35^\circ$

$\widehat{BCG} = \widehat{CED} = 70^\circ$

(ângulos alternos internos)

$\widehat{BGC} = 180^\circ - 70^\circ - 35^\circ = 75^\circ$

Aplicando a lei dos senos ao triângulo

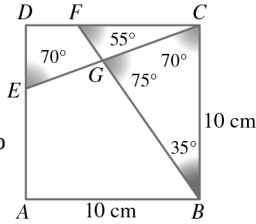
[BCG], temos:

$$\frac{\sin 75^\circ}{10} = \frac{\sin 70^\circ}{\overline{BG}} = \frac{\sin 35^\circ}{\overline{CG}}$$

$$\overline{BG} = \frac{10 \sin 70^\circ}{\sin 75^\circ} \approx 9,728$$

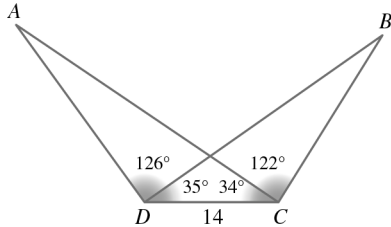
$$\overline{CG} = \frac{10 \sin 35^\circ}{\sin 75^\circ} \approx 5,938$$

$\overline{BG} \approx 9,7 \text{ cm}$ e $\overline{CG} \approx 5,9 \text{ cm}$



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14.

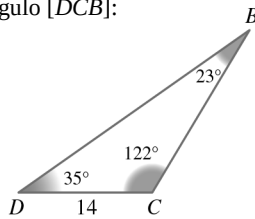


Aplicando a lei dos senos ao triângulo [DCB]:

$\widehat{CBD} = 180^\circ - 122^\circ - 35^\circ = 23^\circ$

$$\frac{\sin 23^\circ}{14} = \frac{\sin 35^\circ}{\overline{BC}}$$

$$\overline{BC} = \frac{14 \times \sin 35^\circ}{\sin 23^\circ} \approx 20,5514$$



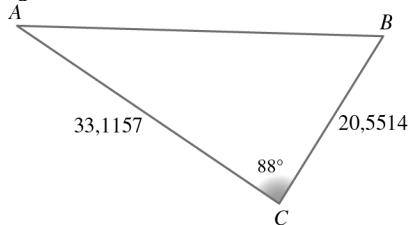
Aplicando a lei dos senos ao triângulo [ADC]:

$\widehat{DAC} = 180^\circ - 126^\circ - 34^\circ = 20^\circ$

$$\frac{\sin 20^\circ}{14} = \frac{\sin 126^\circ}{\overline{AC}}$$

$$\overline{AC} = \frac{14 \times \sin 126^\circ}{\sin 20^\circ} \approx 33,1157$$

Aplicando o Teorema de Carnot ao triângulo [ACB]:



$\widehat{BDA} = 122^\circ - 34^\circ = 88^\circ$

$$\begin{aligned} \overline{AB}^2 &= \overline{AC}^2 + \overline{CB}^2 - 2\overline{AC} \times \overline{CB} \times \cos 88^\circ = \\ &\approx 33,1157^2 + 20,5514^2 - 2 \times 33,1157 \times 20,5514 \times \cos 88^\circ \\ &\approx 1471,506 \end{aligned}$$

$$\overline{AB} \approx \sqrt{1471,506} \approx 38,4 \text{ m}$$

A distância entre A e B é aproximadamente igual a 38,4 m.

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15.1. $\frac{\sin 63^\circ}{6,3} = \frac{\sin B}{7}$

$$\sin B = \frac{7 \times \sin 63^\circ}{6,3} \approx 0,9900$$

Se B é um ângulo agudo,

$$B \approx \sin^{-1}(0,9900) \approx 81,89$$

Se B é um ângulo obtuso,

$$B \approx 180^\circ - 81,89 \approx 98,11$$

Se $B \approx 81,89$, $A \approx 180^\circ - 63^\circ - 81,89^\circ \approx 35,11^\circ$

Se $B \approx 98,11$, $A \approx 180^\circ - 63^\circ - 98,11^\circ \approx 18,89^\circ$

Determinação de $a = \overline{BC}$:

$$\frac{\sin A}{a} = \frac{\sin 63^\circ}{6,3}$$

$$\frac{\sin 35,11^\circ}{a} = \frac{\sin 63^\circ}{6,3} \quad \text{ou} \quad \frac{\sin 18,89^\circ}{a} = \frac{\sin 63^\circ}{6,3}$$

$$a \approx \frac{6,3 \times \sin 35,11^\circ}{\sin 63^\circ} \approx 4,1 \quad \text{ou} \quad a \approx \frac{6,3 \times \sin 18,89^\circ}{\sin 63^\circ} \approx 2,3$$

Conclusão:

$\overline{BC} = 4,1$, $A \approx 35,1^\circ$ e $B \approx 81,9^\circ$ ou

$\overline{BC} = 2,3$, $A \approx 18,9^\circ$ e $B \approx 98,1^\circ$

15.2. $\frac{\sin 55^\circ}{5} = \frac{\sin C}{4}$

$$\sin C = \frac{4 \times \sin 55^\circ}{5} \approx 0,6553$$

Se C é um ângulo agudo,

$$C \approx \sin^{-1}(0,6553) \approx 40,94^\circ$$

Se C é um ângulo obtuso,

$$C = 180^\circ - 40,94^\circ = 139,06^\circ$$

Se $C \approx 40,94^\circ$:

$$B = 180^\circ - (55^\circ + 40,94^\circ) \approx 84,06^\circ$$

Se $C \approx 139,06^\circ$:

$$B = 180^\circ - (55^\circ + 139,06^\circ) \approx 180^\circ - 194,06^\circ < 0$$

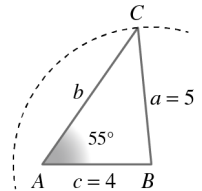
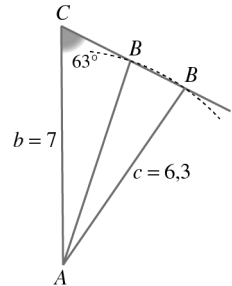
B não pode ser negativo. Logo, o problema tem apenas uma solução: $B \approx 84,06^\circ$

Determinemos $b = \overline{AC}$:

$$\frac{\sin 55^\circ}{5} = \frac{\sin(84,06^\circ)}{b}$$

$$b = \frac{5 \times \sin(84,06^\circ)}{\sin 55^\circ} \approx 6,1$$

$$b \approx 6,1; B \approx 84,1^\circ \text{ e } C \approx 40,9^\circ$$



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16.1. $a = 7$, $c = 4$ e $C = 12^\circ$

$$\frac{\sin 12^\circ}{4} = \frac{\sin A}{7}$$

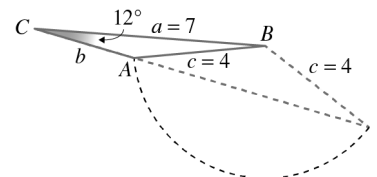
$$\sin A = \frac{7 \times \sin 12^\circ}{4}$$

$$\approx 0,36385$$

Se A é agudo:

$$A \approx \sin^{-1}(0,36385) \approx 21,337$$

$$B \approx 180^\circ - 12^\circ - 21,337^\circ \approx 146,663^\circ$$



$$\frac{\sin 146,663^\circ}{b} = \frac{\sin 12^\circ}{4}$$

$$b = \frac{4 \sin 146,663^\circ}{\sin 12^\circ} \approx 10,6$$

Se A é obtuso:

$$A \approx 180^\circ - 21,337^\circ \approx 158,663^\circ$$

$$B \approx 180^\circ - 12^\circ - 158,663^\circ \approx 9,337^\circ$$

$$\frac{\sin 9,337^\circ}{b} \approx \frac{\sin 12^\circ}{4}$$

$$b \approx \frac{4 \sin 9,337^\circ}{\sin 12^\circ} \approx 3,1$$

$$A = 21,3^\circ; B = 146,7^\circ \text{ e } b = 10,6 \text{ ou}$$

$$A = 158,7^\circ; B = 9,3^\circ \text{ e } b = 3,1$$

Também se pode usar o Teorema de Carnot.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$16 = 49 + b^2 - 2 \times 7 \times b \times \cos 12^\circ \Leftrightarrow$$

$$\Leftrightarrow b^2 - 14 \cos 12^\circ \times b + 33 = 0 \Rightarrow$$

$$\Rightarrow b = 10,573 \vee b \approx 3,121$$

Para estes valores de b , determinam-se A e B .

16.2. $a = 6, c = 20$ e $A = 25^\circ$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 25^\circ}{6} = \frac{\sin C}{20}$$

$$\sin C = \frac{20 \times \sin 25^\circ}{6} \approx 1,4$$

Como $\sin C > 1$, o triângulo $[ABC]$ não existe.

16.3. $a = 8, b = 11$ e $A = 41^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 41^\circ}{8} = \frac{\sin B}{11}$$

$$\sin B = \frac{11 \times \sin 41^\circ}{8} \approx 0,90208$$

Se B é agudo:

$$B \approx \sin^{-1}(0,90208) \approx 64,433^\circ$$

$$C \approx 180^\circ - 41^\circ - 64,433^\circ \approx 74,567^\circ$$

$$\frac{\sin 74,567^\circ}{c} = \frac{\sin 41^\circ}{8}$$

$$c \approx \frac{8 \times \sin 74,567^\circ}{\sin 41^\circ} \approx 11,8$$

Se B é obtuso:

$$B = 180^\circ - 64,433^\circ = 115,567^\circ$$

$$C = 180^\circ - 41^\circ - 115,567^\circ = 23,433^\circ$$

$$\frac{\sin 23,433^\circ}{c} = \frac{\sin 41^\circ}{8}$$

$$c \approx \frac{8 \times \sin 23,433^\circ}{\sin 41^\circ} \approx 4,8$$

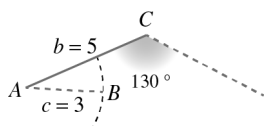
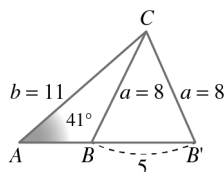
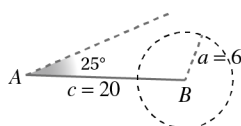
$$B \approx 64,4^\circ; C \approx 74,6^\circ \text{ e } c = 11,8 \text{ ou}$$

$$B \approx 115,6^\circ; C \approx 23,4^\circ \text{ e } c \approx 4,8$$

16.4. $b = 5, c = 3$ e $C = 130^\circ$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 130^\circ}{3} = \frac{\sin B}{5}$$



$$\sin B = \frac{5 \times \sin 130^\circ}{3} \approx 1,28$$

Como $\sin B > 1$, o triângulo $[ABC]$ não existe.

16.5. $b = 7, c = 5$ e $C = 63^\circ$

$$\frac{\sin 63^\circ}{5} = \frac{\sin B}{7}$$

$$\sin B = \frac{7 \sin 63^\circ}{5} \approx 1,25$$

O seno de um ângulo é sempre não superior a 1.

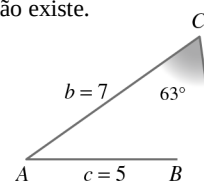
Logo, o triângulo $[ABC]$ não existe.

16.6. $a = 3, b = 7$ e $c = 11$

$c \geq a + b$. Logo, o triângulo $[ABC]$ não existe.

16.7. $A = 110^\circ, B = 70^\circ$ e $c = 10$

$A + B \geq 180^\circ$. Logo, o triângulo $[ABC]$ não existe.



Atividades complementares

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17.1. $B = 180^\circ - 43^\circ - 55^\circ = 82^\circ$

$$\frac{\sin 82^\circ}{10} = \frac{\sin 43^\circ}{a} = \frac{\sin 55^\circ}{c}$$

$$a = \frac{10 \sin 43^\circ}{\sin 82^\circ} \approx 6,9$$

$$c = \frac{10 \sin 55^\circ}{\sin 82^\circ} \approx 8,3$$

$$a \approx 6,9 \text{ cm e } c \approx 8,3 \text{ cm}$$

17.2. $C = 180^\circ - 62^\circ - 56^\circ = 62^\circ$

$$\frac{\sin 56^\circ}{3} = \frac{\sin 62^\circ}{a} = \frac{\sin 62^\circ}{c}$$

$$a = c = \frac{3 \sin 62^\circ}{\sin 56^\circ} \approx 3,2$$

$$a = c \approx 3,2 \text{ cm}$$

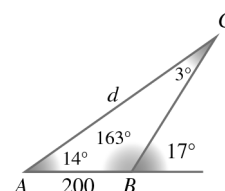
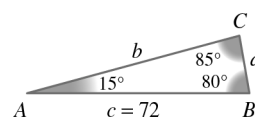
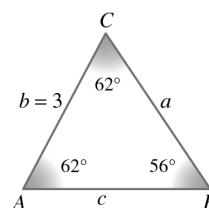
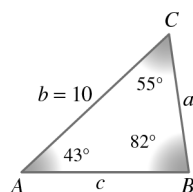
17.3. $A = 180^\circ - 80^\circ - 85^\circ = 15^\circ$

$$\frac{\sin 85^\circ}{72} = \frac{\sin 80^\circ}{b} = \frac{\sin 15^\circ}{a}$$

$$a = \frac{72 \sin 15^\circ}{\sin 85^\circ} \approx 18,7$$

$$b = \frac{72 \sin 80^\circ}{\sin 85^\circ} \approx 71,2$$

$$a \approx 18,7 \text{ cm e } b \approx 71,2 \text{ cm}$$



O desenho não está feito à escala.

18. $\hat{ACB} = 17^\circ - 14^\circ = 3^\circ$

$$\hat{CBA} = 180^\circ - 17^\circ = 163^\circ$$

$$d = \overline{AC}$$

$$\frac{\sin 3^\circ}{200} = \frac{\sin 163^\circ}{d}$$

$$d = \frac{200 \sin 163^\circ}{\sin 3^\circ} \approx 1117$$

A distância entre A e C é aproximadamente 1117 m.

19. $c = \overline{AB} = 10 \text{ m}$

$$C = 105^\circ$$

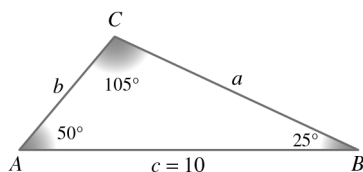
$$A = 2B$$

$$A + B + C = 180^\circ$$

$$2B + B + 105^\circ = 180^\circ$$

$$3B = 75^\circ$$

$$B = 25^\circ \text{ e } A = 50^\circ$$



$$\frac{\sin 105^\circ}{10} = \frac{\sin 50^\circ}{a} = \frac{\sin 25^\circ}{b}$$

$$a = \frac{10 \sin 50^\circ}{\sin 105^\circ} \approx 7,93$$

$$b = \frac{10 \sin 25^\circ}{\sin 105^\circ} \approx 4,38$$

$$P_{[ABC]} \approx (7,93 + 4,38 + 10) \text{ m} \approx 22,3 \text{ m}$$

$$20.1. A = 55^\circ, b = 5 \text{ cm e } c = 4 \text{ cm}$$

$$a^2 = b^2 + c^2 - 2bc \cos A = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos 55^\circ \approx 18,0569$$

$$a \approx \sqrt{18,0569} \approx 4,249$$

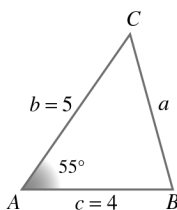
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{18,0569 + 4^2 - 5^2}{2 \times \sqrt{18,0569} \times 4} \approx 0,2664$$

$$B \approx \cos^{-1}(0,2664) \approx 74,5^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{18,0569 + 5^2 - 4^2}{2 \times \sqrt{18,0569} \times 5} \approx 0,6367$$

$$C \approx \cos^{-1}(0,6367) \approx 50,5^\circ$$

$$a \approx 4,2 \text{ cm}; B \approx 74,5^\circ \text{ e } C \approx 50,5^\circ$$



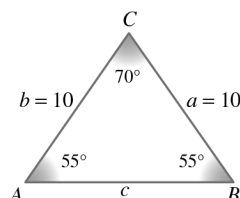
$$20.2. \text{ O triângulo } [ABC] \text{ é isósceles.}$$

$$A = B = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$c^2 = a^2 + b^2 - 2ab \cos C = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 70^\circ \approx 131,5960$$

$$c \approx \sqrt{131,5960} \approx 11,5$$

$$c \approx 11,5 \text{ cm e } A = B = 55^\circ$$



$$20.3. a = 12 \text{ cm, } b = 15 \text{ cm e } c = 17 \text{ cm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{15^2 + 17^2 - 12^2}{2 \times 15 \times 17} = \frac{370}{510} = \frac{37}{51}$$

$$A = \cos^{-1}\left(\frac{37}{51}\right) \approx 43,5^\circ$$

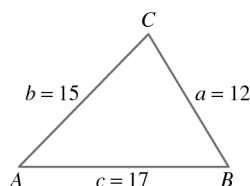
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{12^2 + 17^2 - 15^2}{2 \times 12 \times 17} = \frac{208}{408} = \frac{26}{51}$$

$$B = \cos^{-1}\left(\frac{26}{51}\right) \approx 59,3^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{12^2 + 15^2 - 17^2}{2 \times 12 \times 15} = \frac{80}{360} = \frac{2}{9}$$

$$C = \cos^{-1}\left(\frac{2}{9}\right) \approx 77,2^\circ$$

$$A \approx 43,5^\circ; B \approx 59,3^\circ \text{ e } C \approx 77,2^\circ$$



$$20.4. a = 2,1 \text{ cm; } b = 2 \text{ cm e } c = 2,9 \text{ cm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2^2 + 2,9^2 - 2,1^2}{2 \times 2 \times 2,9} = \frac{8}{11,6} = \frac{20}{29}$$

$$A = \cos^{-1}\left(\frac{20}{29}\right) \approx 46,4^\circ$$

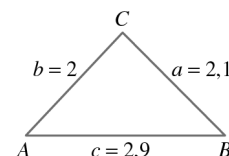
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2,1^2 + 2,9^2 - 2^2}{2 \times 2,1 \times 2,9} = \frac{8,82}{12,18} = \frac{21}{29}$$

$$B = \cos^{-1}\left(\frac{21}{29}\right) \approx 43,6^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2,1^2 + 2^2 - 2,9^2}{2 \times 2,1 \times 2} = 0$$

$$C = \cos^{-1}(0) = 90^\circ$$

$$A \approx 46,4^\circ; B \approx 43,6^\circ \text{ e } C = 90^\circ$$



$$21. d^2 = 250^2 + 300^2 - 2 \times 250 \times 300 \times \cos 85^\circ \approx 139\,429,639$$

$$d \approx \sqrt{139\,429,639} \approx 373,4 \text{ km}$$

A distância entre os dois aviões é de 373,4 km, aproximadamente.

$$22. \text{ Diagonal maior:}$$

$$d_1^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 120^\circ = 20 - 2 \times 8 \times \left(-\frac{1}{2}\right) = 28$$

$$d_1 = \sqrt{28} = 2\sqrt{7}$$

$$\text{Diagonal menor}$$

$$180^\circ - 120^\circ = 60^\circ$$

$$d_2^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 60^\circ = 20 - 2 \times 8 \times \frac{1}{2} = 12$$

$$d_2 = \sqrt{12} = 2\sqrt{3}$$

$$d_1 = 2\sqrt{7} \text{ cm e } d_2 = 2\sqrt{3} \text{ cm}$$

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$$23.1. a = 2, b = 2,8 \text{ e } c = 3,2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2,8^2 + 3,2^2 - 2^2}{2 \times 2,8 \times 3,2} = \frac{14,08}{17,92} = \frac{11}{14}$$

$$A = \cos^{-1}\left(\frac{11}{14}\right) \approx 38,21^\circ$$

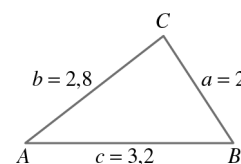
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2^2 + 3,2^2 - 2,8^2}{2 \times 2 \times 3,2} = \frac{6,4}{12,8} = \frac{1}{2}$$

$$B = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2^2 + 2,8^2 - 3,2^2}{2 \times 2 \times 2,8} = \frac{1,6}{11,2} = \frac{1}{7}$$

$$C = \cos^{-1}\left(\frac{1}{7}\right) \approx 81,79^\circ$$

$$A \approx 38,21^\circ; B = 60^\circ \text{ e } C \approx 81,79^\circ$$



23.2. $a = 15$ cm, $b = 8$ cm e $c = 9$ cm

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \\ &= \frac{8^2 + 9^2 - 15^2}{2 \times 8 \times 9} = \\ &= \frac{-80}{144} = -\frac{5}{9}\end{aligned}$$

$$A = \cos^{-1}\left(-\frac{5}{9}\right) \approx 123,75^\circ$$

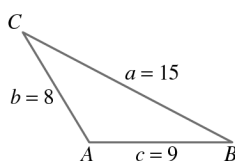
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{15^2 + 9^2 - 8^2}{2 \times 15 \times 9} = \frac{242}{270} = \frac{121}{135}$$

$$B = \cos^{-1}\left(\frac{121}{135}\right) \approx 26,32^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{15^2 + 8^2 - 9^2}{2 \times 15 \times 8} = \frac{208}{240} = \frac{13}{15}$$

$$C = \cos^{-1}\left(\frac{13}{15}\right) \approx 29,93^\circ$$

$$A \approx 123,75^\circ; B \approx 26,32^\circ \text{ e } C \approx 29,93^\circ$$



23.3. $a = 3,3$ cm ; $b = 5,6$ cm e $c = 6,5$ cm

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \\ &= \frac{5,6^2 + 6,5^2 - 3,3^2}{2 \times 5,6 \times 6,5} = \\ &= \frac{62,72}{72,8} = \frac{56}{65}\end{aligned}$$

$$A = \cos^{-1}\left(\frac{56}{65}\right) \approx 30,51^\circ$$

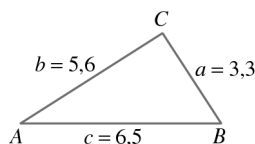
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3,3^2 + 6,5^2 - 5,6^2}{2 \times 3,3 \times 6,5} = \frac{21,78}{42,9} = \frac{33}{65}$$

$$B = \cos^{-1}\left(\frac{33}{65}\right) \approx 59,49^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3,3^2 + 5,6^2 - 6,5^2}{2 \times 3,3 \times 5,6} = 0$$

$$C = \cos^{-1}(0) = 90^\circ$$

$$A \approx 30,51^\circ; B \approx 59,49^\circ \text{ e } C = 90^\circ$$



23.4. $a = 3$ cm, $b = 5$ cm e $c = 7$ cm

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \\ &= \frac{5^2 + 7^2 - 3^2}{2 \times 5 \times 7} = \frac{65}{70} = \frac{13}{14}\end{aligned}$$

$$A = \cos^{-1}\left(\frac{13}{14}\right) = 21,79^\circ$$

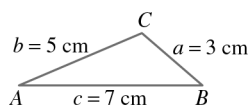
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3^2 + 7^2 - 5^2}{2 \times 3 \times 7} = \frac{33}{42} = \frac{11}{14}$$

$$B = \cos^{-1}\left(\frac{11}{14}\right) = 38,21^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5} = \frac{-15}{30} = -\frac{1}{2}$$

$$C = \cos^{-1}\left(-\frac{1}{2}\right) = 180^\circ - 60^\circ = 120^\circ$$

$$A \approx 21,79^\circ; B \approx 38,21^\circ \text{ e } C = 120^\circ$$



24.1. $a = 5,2$; $b = 3,2$ e $c = 6$

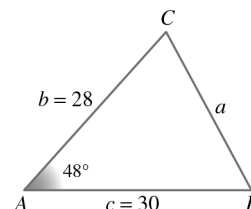
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3,2^2 + 6^2 - 5,2^2}{2 \times 3,2 \times 6} = \frac{19,2}{38,4} = \frac{1}{2}$$

$$A = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$B\hat{A}C = 60^\circ$$

24.2. $\frac{h}{3,2} = \sin 60^\circ \Leftrightarrow h = 3,2 \times \frac{\sqrt{3}}{2} \Leftrightarrow h = \frac{8\sqrt{3}}{5}$

25.1. $A = 48^\circ$, $b = 28$ cm e $c = 30$ cm



$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A = \\ &= 28^2 + 30^2 - 2 \times 28 \times 30 \times \cos 48^\circ \\ &\approx 559,861\end{aligned}$$

$$a \approx \sqrt{559,861} \approx 23,7$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{559,861 + 30^2 - 28^2}{2 \times \sqrt{559,861} \times 30} \approx 0,476$$

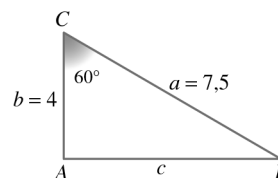
$$B \approx \cos^{-1}(0,476) \approx 61,6^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{559,861 + 28^2 - 30^2}{2 \times \sqrt{559,861} \times 28} \approx 0,335$$

$$C \approx \cos^{-1}(0,335) \approx 70,4^\circ$$

$$a \approx 23,7 \text{ cm}, B \approx 61,6^\circ \text{ e } C \approx 70,4^\circ$$

25.2. $C = 60^\circ$, $a = 7,5$ cm e $b = 4$ cm



$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos 60^\circ = \\ &= 7,5^2 + 4^2 - 2 \times 7,5 \times 4 \times \frac{1}{2} = 42,25\end{aligned}$$

$$c = \sqrt{42,25} = 6,5$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{4^2 + 6,5^2 - 7,5^2}{2 \times 4 \times 6,5} = \frac{2}{52} = \frac{1}{26}$$

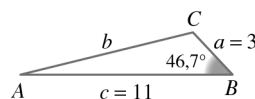
$$A = \cos^{-1}\left(\frac{1}{26}\right) \approx 87,8^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{7,5^2 + 6,5^2 - 4^2}{2 \times 7,5 \times 6,5} = \frac{82,5}{97,5} = \frac{11}{13}$$

$$B = \cos^{-1}\left(\frac{11}{13}\right) \approx 32,2^\circ$$

$$c = 6,5 \text{ cm}; A \approx 87,8^\circ \text{ e } B = 32,2^\circ$$

25.3. $B = 46,7^\circ$; $a = 3$ cm e $c = 11$ cm



$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cos B = \\ &= 3^2 + 11^2 - 2 \times 3 \times 11 \times \cos 46,7^\circ \approx 84,7360\end{aligned}$$

$$b \approx \sqrt{84,7360} \approx 9,2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{84,7360 + 11^2 - 3^2}{2\sqrt{84,7360} \times 11} \approx 0,9715$$

$$A \approx \cos^{-1}(0,9715) \approx 13,7^\circ$$

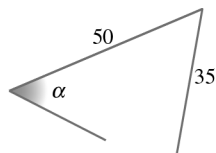
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3^2 + 84,7360 - 11^2}{2 \times 3 \times \sqrt{84,7360}} \approx -0,4936$$

$$C \approx \cos^{-1}(-0,4936) \approx 119,6^\circ$$

$$b = 9,2 \text{ cm}; A \approx 13,7^\circ \text{ e } C \approx 119,6^\circ$$

$$26. \quad \cos \alpha = \frac{50^2 + 41^2 - 35^2}{2 \times 50 \times 41} = \frac{2956}{4100}$$

$$\alpha = \cos^{-1}\left(\frac{2956}{4100}\right) \approx 44^\circ$$



$$27.1. \quad A = 52^\circ, B = 68^\circ \text{ e } c = 15 \text{ cm}$$

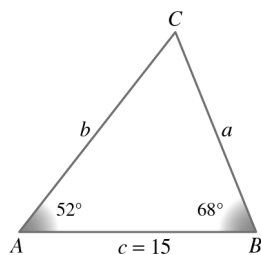
$$C = 180^\circ - 52^\circ - 68^\circ \approx 60^\circ$$

$$\frac{\sin 60^\circ}{15} = \frac{\sin 52^\circ}{a} = \frac{\sin 68^\circ}{b}$$

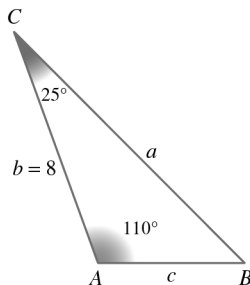
$$a = \frac{15 \sin 52^\circ}{\sin 60^\circ} \approx 13,6$$

$$b = \frac{15 \sin 68^\circ}{\sin 60^\circ} \approx 16,1$$

$$C = 60^\circ; a = 13,6 \text{ cm e } b = 16,1 \text{ cm}$$



$$27.2. \quad B = 180^\circ - 110^\circ - 25^\circ = 45^\circ$$



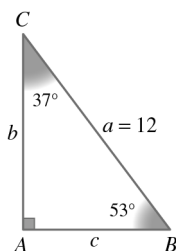
$$\frac{\sin 45^\circ}{8} = \frac{\sin 110^\circ}{a} = \frac{\sin 25^\circ}{c}$$

$$a = \frac{8 \sin 110^\circ}{\sin 45^\circ} \approx 10,6$$

$$c = \frac{8 \sin 25^\circ}{\sin 45^\circ} \approx 4,8$$

$$B = 45^\circ, a \approx 10,6 \text{ cm e } c \approx 4,8 \text{ cm}$$

$$27.3. \quad A = 180^\circ - 37^\circ - 53^\circ = 90^\circ$$



O triângulo [ABC] é retângulo em A.

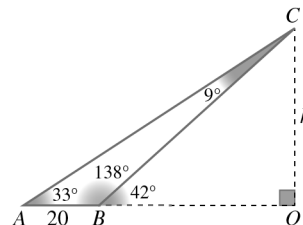
$$\frac{b}{12} = \sin 53^\circ \Leftrightarrow b = 12 \sin 53^\circ \Rightarrow b \approx 9,6$$

$$\frac{c}{12} = \cos(53^\circ) \Leftrightarrow c = 12 \cos(53^\circ) \Rightarrow c \approx 7,2$$

$$A = 90^\circ; b = 9,6 \text{ cm e } c \approx 7,2 \text{ cm}$$

$$28. \quad \hat{C}BA = 180^\circ - 42^\circ = 138^\circ$$

$$\hat{A}CB = 42^\circ - 33^\circ = 9^\circ$$



$$\frac{\sin 9^\circ}{20} = \frac{\sin 33^\circ}{BC}$$

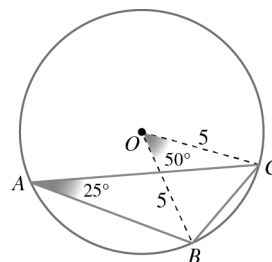
$$BC = \frac{20 \sin 33^\circ}{\sin 9^\circ}$$

$$\frac{h}{BC} = \sin 42^\circ$$

$$h = BC \times \sin 42^\circ = \frac{20 \sin 33^\circ}{\sin 9^\circ} \times \sin 42^\circ \approx 46,6$$

O prédio tem 46,6 m de altura.

29.



$$\widehat{BC} = 2 \times 25^\circ = 50^\circ$$

$$\widehat{BOC} = 50^\circ$$

$$BO = OC = 5 \text{ cm}$$

$$\overline{BC}^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 50^\circ \approx 17,8606$$

$$\overline{BC} \approx \sqrt{17,8606} \approx 4,2$$

$$\overline{BC} \approx 4,2 \text{ cm}$$

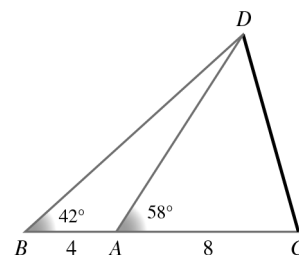
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$$30. \quad \hat{A}DB = 58^\circ - 42^\circ = 16^\circ$$

Pela lei dos senos no triângulo [BAD]:

$$\frac{\sin 16^\circ}{4} = \frac{\sin 42^\circ}{AD}$$

$$\overline{AD} = \frac{4 \sin 42^\circ}{\sin 16^\circ} \approx 9,7103$$



Aplicando a lei dos

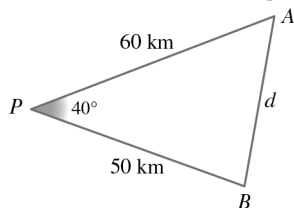
cosenos ao triângulo [ACD]:

$$\overline{DC}^2 \approx 8^2 + (9,7103)^2 - 2 \times 8 \times 9,7103 \times \cos 58^\circ \approx 75,9591$$

$$\overline{DC} \approx \sqrt{75,9591} \approx 8,72$$

O poste mede 8,72 m, aproximadamente.

31. $2\text{ h } 30\text{ min} = 2,5\text{ h}$
 $2,5 \times 24 = 60 \rightarrow$ O barco A percorreu 60 km.
 $2,5 \times 20 = 50 \rightarrow$ O barco B percorreu 50 km.



$$d^2 = 60^2 + 50^2 - 2 \times 60 \times 50 \times \cos 40^\circ \approx 1503,733$$

$$d \approx \sqrt{1503,733} \approx 38,778$$

O barco percorreu 38,778 km a uma velocidade de 24 km/h.

$$V = \frac{e}{t}, \text{ logo } 24 = \frac{38,778}{t} \Leftrightarrow t = \frac{38,778}{24}$$

$$t \approx 1,616, \text{ pelo que } t \approx 1\text{ h } 37\text{ min} \quad 0,616 \times 60 = 36,96$$

O barco demorou 1h 37 min.

32. $\overline{FB} = a$
 $\widehat{AFB} = 180^\circ - 59^\circ - 83^\circ = 38^\circ$

$$\frac{\sin 59^\circ}{a} = \frac{\sin 38^\circ}{5}$$

$$a = \frac{5 \sin 59^\circ}{\sin 38^\circ} \approx 6,9614$$

Seja $\beta = \widehat{HBA}$.

$$\cos \beta = \frac{3^2 + 5^2 - 4^2}{2 \times 3 \times 5} = \frac{3}{5}$$

$$\beta = \cos^{-1}\left(\frac{3}{5}\right) \approx 53,1301^\circ$$

Seja $\alpha = \widehat{FBH}$.

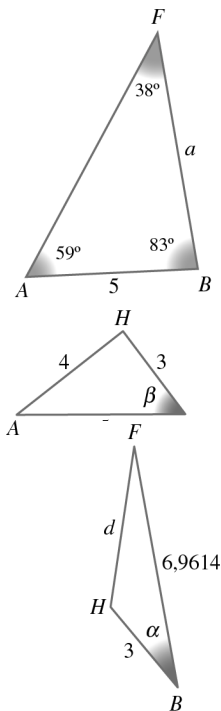
$$\alpha = 83^\circ - 53,1301^\circ \approx 29,8699^\circ$$

$$d = \overline{HF}$$

$$d^2 \approx 3^2 + 6,9614^2 - 2 \times 3 \times 6,9614 \times \cos 29,8699^\circ \approx 21,2413$$

$$d \approx \sqrt{21,2413} \approx 4,6$$

O helicóptero está a cerca de 4,6 km do incêndio.

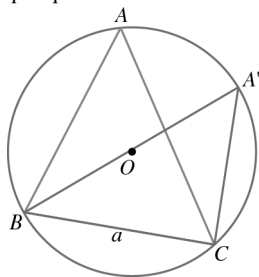


33. Seja $[ABC]$ um triângulo qualquer inscrito numa circunferência de centro O e raio r e seja A' o ponto da circunferência tal que $[BA']$ é um diâmetro.

Então, o triângulo $[A'BC]$ é um retângulo em C pelo que $\frac{\overline{BC}}{\overline{BA'}} = \sin(\widehat{BA'C})$.

Como $\overline{BC} = a$, $\overline{BA'} = 2r$ e $\widehat{BA'C} = \widehat{BAC} = \frac{1}{2}\widehat{BC}$, temos

que $\frac{a}{2r} = \sin A$, ou seja, $\frac{a}{\sin A} = 2r$.



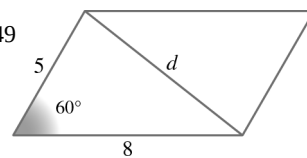
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1. $d^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60^\circ =$

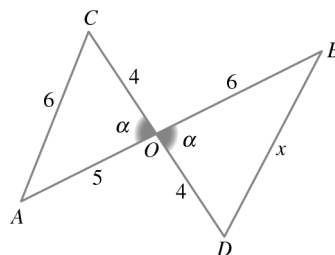
$$= 64 + 25 - 2 \times 40 \times \frac{1}{2} = 49$$

$$d = \sqrt{49} = 7$$

Resposta: (B)



2. Se $\alpha = \widehat{COA}$, então $\widehat{DOB} = \alpha$ (ângulos verticalmente opostos). Seja $x = \overline{BD}$.



$$\cos \alpha = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = \frac{5}{40} = \frac{1}{8}$$

$$x^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \cos \alpha = 36 + 16 - 48 \times \frac{1}{8} = 46$$

$$\overline{DB} = x = \sqrt{46} \text{ cm}$$

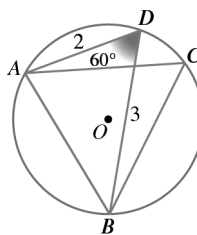
Resposta: (C)

3. $\frac{\sin 30^\circ}{2\sqrt{2}} = \frac{\sin 45^\circ}{x}$

$$x = \frac{2\sqrt{2} \times \sin 45^\circ}{\sin 30^\circ} = \frac{2\sqrt{2} \times \frac{\sqrt{2}}{2}}{\frac{1}{2}} = 2 \times 2 = 4$$

Resposta: (A)

- 4.



$[ABC]$ é um triângulo equilátero, logo $\widehat{AB} = \frac{360^\circ}{3} = 120^\circ$.

Assim, $\widehat{ADC} = \frac{120^\circ}{2} = 60^\circ$.

Pela lei dos cossenos:

$$\overline{AB}^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 60^\circ = 4 + 9 - 6 = 7$$

Logo, $\overline{AB} = \sqrt{7} \text{ cm}$.

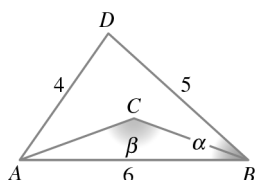
Resposta: (B)

5. $\frac{\sin 30^\circ}{2} = \frac{\sin x}{3}$

$$\sin x = \frac{3 \sin 30^\circ}{2} = \frac{3 \times \frac{1}{2}}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

Resposta: (D)

- 6.



$$\widehat{BAC} = \widehat{CBA} = \frac{\alpha}{2}$$

$$\frac{\alpha}{2} + \frac{\alpha}{2} + \beta = \pi \Leftrightarrow \alpha + \beta = \pi \Leftrightarrow \beta = \pi - \alpha$$

$$\cos \alpha = \frac{6^2 + 5^2 - 4^2}{2 \times 6 \times 5} = \frac{45}{60} = \frac{3}{4}$$

$$\cos \beta = \cos(\pi - \alpha) = -\cos \alpha = -\frac{3}{4}$$

Resposta: (D)

$$x = \sqrt{6,76} = 2,6$$

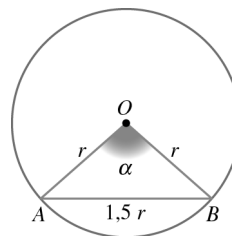
$$\overline{AB} = 1,4 + 2,6 = 4$$

$$y^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos 60^\circ = 9 + 16 - 12 = 13$$

$$y = \sqrt{13}$$

$$\overline{CB} = \sqrt{13} \text{ cm}$$

10.



$$(1,5r)^2 = r^2 + r^2 - 2 \times r \times r \times \cos \alpha$$

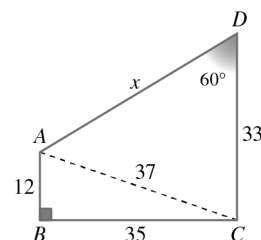
$$2,25r^2 = 2r^2 - 2r^2 \cos \alpha$$

$$2r^2 \cos \alpha = 2r^2 - 2,25r^2$$

$$\cos \alpha = \frac{-0,25r^2}{2r^2} \Leftrightarrow$$

$$\Leftrightarrow \cos \alpha = -\frac{0,25}{2} \Leftrightarrow \cos \alpha = -\frac{1}{8}$$

11. O triângulo [ABC] é retângulo em C.



$$\overline{AC}^2 = 12^2 + 35^2$$

$$\overline{AC} = \sqrt{144 + 1225} = \sqrt{1369} = 37$$

Pela lei dos cossenos, se $\overline{AD} = x$ cm:

$$37^2 = x^2 + 33^2 - 2x + 33 \times \cos 60^\circ \Leftrightarrow$$

$$\Leftrightarrow x^2 - 33x - 280 = 0 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{33 \pm \sqrt{(-33)^2 - 4 \times (-280)}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = 40 \vee x = -7$$

Como $x > 0$, temos $\overline{AD} = 40$ cm.

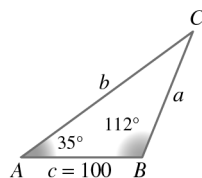
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7. $C = 180^\circ - 35^\circ - 112^\circ = 33^\circ$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

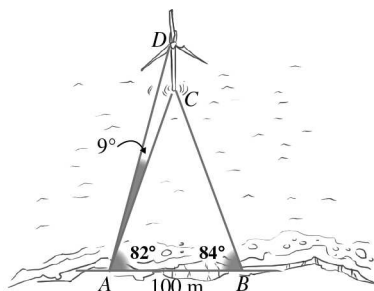
$$\frac{\sin 33^\circ}{100} = \frac{\sin 35^\circ}{a}$$

$$a = \frac{100 \times \sin 35^\circ}{\sin 33^\circ} \approx 105,3$$



A distância de B a C é de 105,3 m, aproximadamente.

8.



$$\widehat{BCA} = 180^\circ - 82^\circ - 84^\circ = 14^\circ$$

$$\frac{\sin 14^\circ}{100} = \frac{\sin 84^\circ}{\overline{AC}} \quad (\text{Lei dos senos no triângulo [ABC]})$$

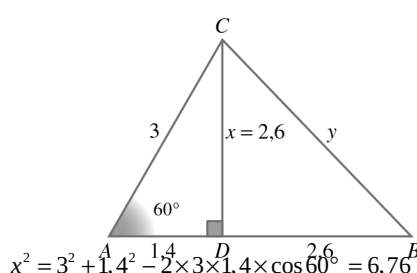
$$\overline{AC} = \frac{100 \sin 84^\circ}{\sin 14^\circ} \approx 411,092$$

$$\frac{\overline{DC}}{\overline{AC}} = \tan 9^\circ \quad (\text{O triângulo [ACD] é retângulo em C.})$$

$$\frac{\overline{DC}}{411,092} \approx \tan 9^\circ \Rightarrow \overline{DC} \approx 411,092 \times \tan 9^\circ \Rightarrow \overline{DC} \approx 65$$

A torre tem 65 m de altura, aproximadamente.

9. Seja $x = \overline{CD}$ e $y = \overline{CB}$.



$$x^2 = 3^2 + 1,4^2 - 2 \times 3 \times 1,4 \times \cos 60^\circ = 6,76$$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

Atividade inicial 2

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- 1.1. C 1.2. E 1.3. F
1.4. B 1.5. D 1.6. D
2. Rotação de centro O e amplitude 60° de sentido positivo ou rotação de centro O e amplitude 300° de sentido negativo

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- 1.1. C
1.2. $\alpha = 90^\circ$ ou $\alpha = -270^\circ$

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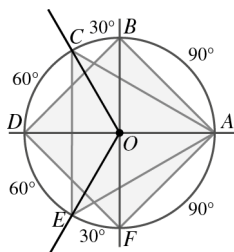
- 2.1. a) $225^\circ : 45^\circ = 5$
Lado extremidade: $\dot{O}F$
b) $135^\circ : 45^\circ = 3$
Lado extremidade: $\dot{O}F$
- 2.2. a) $1575^\circ = 135^\circ + 4 \times 360^\circ$
$$\begin{array}{r} 1575 \\ \underline{135} \quad 4 \\ 135 \end{array}$$

Lado extremidade: $\dot{O}D$
b) $-1170^\circ = -90^\circ - 3 \times 360^\circ$
$$\begin{array}{r} 1170^\circ \\ \underline{90^\circ} \quad 3 \\ 90^\circ \end{array}$$

Lado extremidade: $\dot{O}G$

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3. $120^\circ - 90^\circ = 30^\circ$



- 3.1. $1170^\circ = 90^\circ + 3 \times 360^\circ$
O transformado do ponto A é o ponto B.

$$\begin{array}{r} 1170 \\ \underline{90} \quad 3 \\ 90 \end{array}$$

- 3.2. $900^\circ = 180^\circ + 2 \times 360^\circ$
O transformado do ponto A é o ponto D.

$$\begin{array}{r} 900 \\ \underline{180} \quad 2 \\ 180 \end{array}$$

- 3.3. $600^\circ = 240^\circ + 360^\circ$
 $240^\circ = 180^\circ + 60^\circ$
O transformado do ponto A é o ponto E.

$$\begin{array}{r} 600 \\ \underline{240} \quad 1 \\ 240 \end{array}$$

- 3.4. $-1530^\circ = -90^\circ - 4 \times 360^\circ$
O transformado do ponto A é o ponto F.

$$\begin{array}{r} 1530 \\ \underline{90} \quad 4 \\ 90 \end{array}$$

- 3.5. $-840^\circ = -120^\circ - 2 \times 360^\circ$
O transformado do ponto A é o ponto E.

$$\begin{array}{r} 840 \\ \underline{120} \quad 2 \\ 120 \end{array}$$

- 3.6. $-1080^\circ = -3 \times 360^\circ$
O transformado do ponto A é o ponto A.

$$\begin{array}{r} 1080 \\ \underline{0} \quad 3 \\ 0 \end{array}$$

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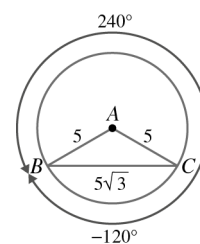
4. $\alpha = \hat{BAC}$

$$\cos \alpha = \frac{5^2 + 5^2 - (5\sqrt{3})^2}{2 \times 5 \times 5} = \frac{50 - 75}{50} = -\frac{25}{50} = -\frac{1}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 180^\circ - 60^\circ = 120^\circ$$

$$360^\circ - 120^\circ = 240^\circ$$

$$\alpha = -120^\circ \text{ e } \alpha = 240^\circ$$



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- 5.1.
 α é do 3.º quadrante
 $\sin \alpha < 0$ e $\cos \alpha < 0$

- 5.2.
 α é do 4.º quadrante
 $\sin \alpha < 0$ e $\cos \alpha > 0$

- 5.3.
 α é do 2.º quadrante
 $\sin \alpha > 0$ e $\cos \alpha < 0$

- 5.4.
 α é do 2.º quadrante
 $\sin \alpha > 0$ e $\cos \alpha < 0$

6. α é do 1.º quadrante
6.1. $\alpha \in 1.^\circ Q$; $\sin \alpha > 0$
 $\alpha + 180^\circ \in 3.^\circ Q$; $\cos(\alpha + 180^\circ) < 0$
 $\sin \alpha + \cos(\alpha + 180^\circ) < 0$
O sinal é negativo.
6.2. $\alpha + 90^\circ \in 2.^\circ Q$; $\cos(\alpha + 90^\circ) < 0$
 $\alpha - 90^\circ \in 4.^\circ Q$; $\tan(\alpha - 90^\circ) < 0$
 $\cos(\alpha + 90^\circ) \times \tan(\alpha - 90^\circ) > 0$
O sinal é positivo.

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

6.3. $\alpha - 90^\circ \in 4.^\circ Q$; $\cos(\alpha - 90^\circ) > 0$

$\alpha + 90^\circ \in 2.^\circ Q$; $\sin(\alpha + 90^\circ) > 0$

$\cos(\alpha - 90^\circ) + \sin(\alpha + 90^\circ) > 0$.

O sinal é positivo.

6.4. $\alpha + 180^\circ \in 3.^\circ Q$; $\tan(\alpha + 180^\circ) > 0$

$\alpha - 90^\circ \in 4.^\circ Q$; $-\sin(\alpha - 90^\circ) > 0$

$\tan(\alpha + 180^\circ) - \sin(\alpha - 90^\circ) > 0$

O sinal é positivo.

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7.1. $P(\cos 30^\circ, \sin 30^\circ)$ ou $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$M(1, \tan 30^\circ)$ ou $M\left(1, \frac{\sqrt{3}}{3}\right)$

Q é a imagem de P pela reflexão central de centro O . Logo,

$Q\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

7.2. $\widehat{AOQ} = 180^\circ + 30^\circ = 210^\circ$

Logo, como Q tem coordenadas $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, então

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$ e $\sin 210^\circ = -\frac{1}{2}$.

7.3. $R(\cos 150^\circ, \sin 150^\circ)$

$\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

$\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$

$R\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

7.4. S é a imagem de R pela reflexão central de centro O .

Logo, S tem coordenadas $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

Portanto, como $\widehat{POS} = 360^\circ - 30^\circ = 330^\circ$, temos

$\sin 330^\circ = -\frac{1}{2}$ e $\cos 330^\circ = \frac{\sqrt{3}}{2}$.

7.5. N é a imagem de M pela reflexão de eixo Ox .

Logo, $N\left(1, -\frac{\sqrt{3}}{3}\right)$.

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8.1. $765^\circ = 45^\circ + 2 \times 360$

$765^\circ = (45^\circ, 2)$

$\sin 765^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 765^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 765^\circ = \tan 45^\circ = 1$

8.2. $780^\circ = 60^\circ + 2 \times 360^\circ$

$780^\circ = (60^\circ, 2)$

$\sin 780^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 780^\circ = \cos 60^\circ = \frac{1}{2}$

$\tan 780^\circ = \tan 60^\circ = \sqrt{3}$

8.3. $1470^\circ = 30^\circ + 4 \times 360^\circ$

$1470^\circ = (30^\circ, 4)$

$\sin 1470^\circ = \sin 30^\circ = \frac{1}{2}$

$\cos 1470^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 1470^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$

9.1. $495^\circ = 135^\circ + 360^\circ$

$495^\circ = (135^\circ, 1)$

$\sin 495^\circ = \sin 135^\circ = \sin(180^\circ - 45^\circ) =$

$= \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 495^\circ = \cos 135^\circ = \cos(180^\circ - 45^\circ) =$

$= -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

9.2. $840^\circ = 120^\circ + 2 \times 360^\circ$

$840^\circ = (120^\circ, 2)$

$\sin 840^\circ = \sin 120^\circ = \sin(180^\circ - 60^\circ) =$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 840^\circ = \cos 120^\circ = \cos(180^\circ - 60^\circ) =$

$= -\cos 60^\circ = -\frac{1}{2}$

9.3. $1230^\circ = 150^\circ + 3 \times 360^\circ$

$1230^\circ = (150^\circ, 3)$

$\sin 1230^\circ = \sin 150^\circ = \sin(180^\circ - 30^\circ) =$

$= \sin 30^\circ = \frac{1}{2}$

$\cos 1230^\circ = \cos 150^\circ = \cos(180^\circ - 30^\circ) =$

$= -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

9.4. $1170^\circ = 90^\circ + 3 \times 360^\circ$

$1170^\circ = (90^\circ, 3)$

$\sin 1170^\circ = \sin 90^\circ = 1$

$\cos 1170^\circ = \cos 90^\circ = 0$

$\frac{1470}{30} \left| \frac{360}{4} \right.$

$\frac{840}{120} \left| \frac{360}{2} \right.$

$\frac{1230}{150} \left| \frac{360}{3} \right.$

$\frac{1170}{90} \left| \frac{360}{3} \right.$

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10.1.

Graus Rad

180 — π

135 — x

$x = \frac{135\pi}{180} = \frac{3\pi}{4}$

$135^\circ = \frac{3\pi}{4} \text{ rad}$

$\frac{780}{60} \left| \frac{360}{2} \right.$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

10.2.

Graus		Rad
180	—	π
270	—	x
$x = \frac{270\pi}{180} = \frac{3\pi}{2}$		
$180^\circ = \frac{3\pi}{2} \text{ rad}$		

10.3.

Graus		Rad
180	—	π
150	—	x
$x = \frac{150\pi}{180} = \frac{5\pi}{6}$		
$150^\circ = \frac{5\pi}{6} \text{ rad}$		

10.4.

Graus		Rad
180	—	π
330	—	x
$x = \frac{330\pi}{180} = \frac{11\pi}{6}$		
$330^\circ = \frac{11\pi}{6}$		

10.5.

Graus		Rad
180	—	π
450	—	x
$x = \frac{450\pi}{180} = \frac{5\pi}{2}$		
$450^\circ = \frac{5\pi}{2} \text{ rad}$		

10.6.

Graus		Rad
180	—	π
252	—	x
$x = \frac{252\pi}{180} = \frac{7\pi}{5}$		
$252^\circ = \frac{7\pi}{5}$		

10.7.

Graus		Rad
180	—	π
202,5	—	x
$x = \frac{202,5\pi}{180} = \frac{9\pi}{8}; 202,5^\circ = \frac{9\pi}{8} \text{ rad}$		

10.8. $337^\circ 30' = 337,5^\circ$

Graus		Rad
180	—	π
337,5	—	x
$x = \frac{337,5\pi}{180} = \frac{15\pi}{8}$		
$337,5^\circ = \frac{15\pi}{8}$		

10.9. $39^\circ 22' 30'' = \left(39 + \frac{22}{60} + \frac{30}{60^2}\right)^\circ = 39,375^\circ$

Graus		Rad
180	—	π
39,375	—	x
$x = \frac{39,375\pi}{180} = \frac{7\pi}{32}$		
$39^\circ 22' 30'' = \frac{7\pi}{32} \text{ rad}$		

11.1.

Graus		Rad
180	—	π
x	—	$\frac{2\pi}{3}$

$$x = \frac{180 \times \frac{2\pi}{3}}{\pi} = 120$$

$$\frac{2\pi}{3} \text{ rad} = 120^\circ$$

11.2.

Graus		Rad
180	—	π
x	—	$\frac{5\pi}{6}$

$$x = \frac{180 \times \frac{5\pi}{6}}{\pi} = 150$$

$$\frac{5\pi}{6} \text{ rad} = 150^\circ$$

11.3.

Graus		Rad
180	—	π
x	—	$\frac{5\pi}{4}$

$$x = \frac{180 \times \frac{5\pi}{4}}{\pi} = 225$$

$$\frac{5\pi}{4} \text{ rad} = 225^\circ$$

11.4.

Graus		Rad
180	—	π
x	—	$\frac{5\pi}{8}$

$$x = \frac{180 \times \frac{5\pi}{8}}{\pi} = 112,5; \frac{5\pi}{8} \text{ rad} = 112,5^\circ$$

12.1.

Graus		Rad
180	—	π
x	—	8

$$x = \frac{180 \times 8}{\pi} \approx 458,366 \text{ 24}$$

$$0,366 \text{ 24} \times 60 \approx 21,9744$$

$$0,9744 \times 60 \approx 58$$

$$8 \text{ rad} \approx 458^\circ 21' 58''$$

12.2.

Graus		Rad
180	—	π
x	—	$\frac{8\pi}{7}$

$$x = \frac{180 \times \frac{8\pi}{7}}{\pi} \approx 205,714\ 29$$

$$0,714\ 29 \times 60 \approx 42,8574$$

$$0,857 \times 60 \approx 51$$

$$\frac{8\pi}{7} \text{ rad} \approx 205^\circ\ 42'\ 51''$$

12.3.

Graus		Rad
180	—	π
x	—	$\frac{5\pi}{11}$

$$x = \frac{180 \times \frac{5\pi}{11}}{\pi} \approx 81,818\ 18$$

$$0,818\ 18 \times 60 \approx 49,0908$$

$$0,0908 \times 60 \approx 5$$

$$\frac{5\pi}{11} \text{ rad} \approx 81^\circ\ 49'\ 5''$$

12.4.

Graus		Rad
180	—	π
x	—	2,4

$$x = \frac{180 \times 2,4}{\pi} \approx 137,509\ 87$$

$$0,509\ 87 \times 60 \approx 30,5922$$

$$0,5922 \times 60 \approx 36$$

$$2,4 \text{ rad} \approx 137^\circ\ 30'\ 36''$$

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13.1. $\sin \alpha = -\frac{1}{3}$ e $\alpha \in 3.^\circ Q$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\left(-\frac{1}{3}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow$$

$$\Leftrightarrow \cos^2 \alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2 \alpha = \frac{8}{9} \Leftrightarrow$$

$$\Leftrightarrow \cos \alpha = \pm \frac{\sqrt{8}}{3}$$

Como $\alpha \in 3.^\circ Q$, $\cos \alpha < 0$. Logo, $\cos \alpha = -\frac{2\sqrt{2}}{3}$.

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = \frac{3}{3 \times 2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\cos \alpha = -\frac{2\sqrt{2}}{3} \text{ e } \tan \alpha = \frac{\sqrt{2}}{4}$$

13.2. $\cos \alpha = \frac{1}{5}$ e $\alpha \in 4.^\circ Q$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \left(\frac{1}{5}\right)^2 = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{1}{25} \Leftrightarrow$$

$$\Leftrightarrow \sin^2 \alpha = \frac{24}{25} \Leftrightarrow \sin \alpha = \pm \frac{\sqrt{24}}{5}$$

Como $\alpha \in 4.^\circ Q$, $\sin \alpha < 0$. Logo, $\sin \alpha = -\frac{2\sqrt{6}}{5}$.

$$\tan \alpha = \frac{-\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = -2\sqrt{6}$$

$$\sin \alpha = -\frac{2\sqrt{6}}{5} \text{ e } \tan \alpha = -2\sqrt{6}$$

13.3. $\tan \alpha = -\frac{24}{7}$ e $\alpha \in 2.^\circ Q$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + \left(-\frac{24}{7}\right)^2 = \frac{1}{\cos^2 \alpha} \Leftrightarrow 1 + \frac{576}{49} = \frac{1}{\cos^2 \alpha} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\cos^2 \alpha} = \frac{625}{49} \Leftrightarrow \cos^2 \alpha = \frac{49}{625} \Leftrightarrow$$

$$\Leftrightarrow \cos \alpha = \pm \frac{7}{25}$$

Como $\alpha \in 2.^\circ Q$, $\cos \alpha < 0$. Logo, $\cos \alpha = -\frac{7}{25}$.

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \frac{49}{625} = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{49}{625} \Leftrightarrow$$

$$\Leftrightarrow \sin^2 \alpha = \frac{576}{625} \Leftrightarrow \sin \alpha = \pm \frac{24}{25}$$

Como $\alpha \in 2.^\circ Q$, $\sin \alpha > 0$. Logo, $\sin \alpha = \frac{24}{25}$.

$$\cos \alpha = -\frac{7}{25} \text{ e } \sin \alpha = \frac{24}{25}$$

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14. Seja $\alpha = \widehat{BOP}$.

$P(\cos \alpha, \sin \alpha)$ e $A(1, \tan \alpha)$

Portanto, $\sin \alpha = -\frac{1}{3}$ e $\alpha \in 4.^\circ Q$.

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\left(-\frac{1}{3}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2 \alpha = \frac{8}{9} \Leftrightarrow$$

$$\Leftrightarrow \cos \alpha = \pm \frac{2\sqrt{2}}{3}$$

Como $\alpha \in 4.^\circ Q$, então $\cos \alpha = \frac{2\sqrt{2}}{3}$.

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = -\frac{3}{3 \times 2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$A\left(1, -\frac{\sqrt{2}}{4}\right); \quad \overline{OC} = \overline{BA} = \frac{\sqrt{2}}{4}$$

O ponto C tem coordenadas $\left(0, \frac{\sqrt{2}}{4}\right)$.

$$\begin{aligned} 15.1. \quad \frac{\sin^2 x}{1 - \cos x} &= \frac{1 - \cos^2 x}{1 - \cos x} = \\ &= \frac{(1 - \cos x)(1 + \cos x)}{(1 - \cos x)} = \\ &= 1 + \cos x \end{aligned}$$

$$\begin{aligned} 15.2. \quad \frac{1 + \sin x}{\cos x} &= \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)} = \\ &= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} = \\ &= \frac{\cos^2 x}{\cos x(1 - \sin x)} = \frac{\cos x}{1 - \sin x} \end{aligned}$$

$$\begin{aligned} 15.3. \quad \frac{1 - \tan^2 x}{1 + \tan^2 x} &= \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} = \\ &= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \cos^2 x = \\ &= \cos^2 x - \sin^2 x \end{aligned}$$

$$\begin{aligned} 15.4. \quad \frac{\tan^3 x}{\cos^2 x} - \tan^3 x &= \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) = \\ &= \tan^3 x \times \frac{1 - \cos^2 x}{\cos^2 x} = \tan^3 x \times \frac{\sin^2 x}{\cos^2 x} = \\ &= \tan^3 x \times \tan^2 x = \\ &= \tan^5 x \end{aligned}$$

$$\begin{aligned} 15.5. \quad \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} &= \frac{(1 - \cos x)^2 + \sin^2 x}{\sin x(1 - \cos x)} = \\ &= \frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1 - \cos x)} = \\ &= \frac{1 - 2\cos x + 1}{\sin x(1 - \cos x)} = \\ &= \frac{2 - 2\cos x}{\sin x(1 - \cos x)} = \\ &= \frac{2(1 - \cos x)}{\sin x(1 - \cos x)} = \frac{2}{\sin x} \end{aligned}$$

$$16.5. \quad \cos\left(\frac{3\pi}{4}\right) = \cos\left(\frac{4\pi}{4} - \frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\begin{aligned} 16.6. \quad \tan\left(-\frac{11\pi}{6}\right) &= -\tan\frac{11\pi}{6} = -\tan\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) = \\ &= -\tan\left(2\pi - \frac{\pi}{6}\right) = -\tan\left(-\frac{\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} 16.7. \quad \sin\left(-\frac{7\pi}{4}\right) &= -\sin\frac{7\pi}{4} = -\sin\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) = \\ &= -\sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\left(-\frac{\pi}{4}\right) = \\ &= \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \end{aligned}$$

$$16.8. \quad \cos\frac{5\pi}{6} = \cos\left(\frac{6\pi}{6} - \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$16.9. \quad \tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$$

$$\begin{aligned} 16.10. \quad \sin\left(\frac{5\pi}{3}\right) &= \sin\left(\frac{6\pi}{3} - \frac{\pi}{3}\right) = \sin\left(2\pi - \frac{\pi}{3}\right) = \\ &= \sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \end{aligned}$$

$$16.11. \quad \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\begin{aligned} 16.12. \quad \tan\left(-\frac{2\pi}{3}\right) &= -\tan\left(\frac{2\pi}{3}\right) = -\tan\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) = \\ &= -\tan\left(\pi - \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \end{aligned}$$

$$\begin{aligned} 16.13. \quad \sin\left(-\frac{5\pi}{6}\right) &= -\sin\left(\frac{5\pi}{6}\right) = -\sin\left(\frac{6\pi}{6} - \frac{\pi}{6}\right) = \\ &= -\sin\left(\pi - \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 16.14. \quad \cos\left(\frac{11\pi}{6}\right) &= \cos\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) = \cos\left(2\pi - \frac{\pi}{6}\right) = \\ &= \cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \end{aligned}$$

$$16.15. \quad \tan\left(-\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$$

$$\begin{aligned} 16.16. \quad \sin\left(-\frac{4\pi}{3}\right) &= -\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) = \\ &= -\sin\left(\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} 17.1. \quad \cos\left(\frac{5\pi}{4}\right) - \sin\left(-\frac{3\pi}{4}\right) - 2\tan\left(\frac{4\pi}{3}\right) &= \\ &= \cos\left(\frac{4\pi}{4} + \frac{\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right) - 2\tan\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) = \\ &= \cos\left(\pi + \frac{\pi}{4}\right) + \sin\left(\frac{4\pi}{4} - \frac{\pi}{4}\right) - 2\tan\left(\pi + \frac{\pi}{3}\right) = \\ &= -\cos\frac{\pi}{4} + \sin\left(\pi - \frac{\pi}{4}\right) - 2\tan\left(\frac{\pi}{3}\right) = \\ &= -\frac{\sqrt{2}}{2} + \sin\left(\frac{\pi}{4}\right) - 2\sqrt{3} = \\ &= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 2\sqrt{3} = -2\sqrt{3} \end{aligned}$$

$$16.1. \quad \sin\frac{7\pi}{6} = \sin\left(\frac{6\pi}{6} + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\begin{aligned} 16.2. \quad \cos\left(-\frac{5\pi}{3}\right) &= \cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{6\pi}{3} - \frac{\pi}{3}\right) = \\ &= \cos\left(2\pi - \frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 16.3. \quad \tan\left(\frac{4\pi}{3}\right) &= \tan\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \\ &= \tan\frac{\pi}{3} = \sqrt{3} \end{aligned}$$

$$\begin{aligned} 16.4. \quad \sin\left(-\frac{5\pi}{4}\right) &= -\sin\frac{5\pi}{4} = -\sin\left(\frac{4\pi}{4} + \frac{\pi}{4}\right) = \\ &= -\sin\left(\pi + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{aligned}$$

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$$\begin{aligned}
 17.2. \quad & \frac{\cos\left(\frac{7\pi}{6}\right) - \sin\left(-\frac{5\pi}{3}\right)}{\tan\left(\frac{13\pi}{6}\right)} = \frac{\cos\left(\frac{6\pi}{6} + \frac{\pi}{6}\right) + \sin\left(\frac{6\pi}{3} - \frac{\pi}{3}\right)}{\tan\left(\frac{12\pi}{6} + \frac{\pi}{6}\right)} = \\
 & = \frac{\cos\left(\pi + \frac{\pi}{6}\right) + \sin\left(2\pi - \frac{\pi}{3}\right)}{\tan\left(2\pi + \frac{\pi}{6}\right)} = \\
 & = \frac{\cos\left(\pi + \frac{\pi}{6}\right) + \sin\left(2\pi - \frac{\pi}{3}\right)}{\tan\left(2\pi + \frac{\pi}{6}\right)} = \\
 & = \frac{-\cos\left(\frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{3}\right)}{\tan\left(\frac{\pi}{6}\right)} = \frac{-\frac{\sqrt{3}}{2} - \sin\left(\frac{\pi}{3}\right)}{\frac{\sqrt{3}}{3}} = \\
 & = \frac{-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{3}} = -\frac{\sqrt{3}}{\frac{\sqrt{3}}{3}} = \\
 & = -\frac{3\sqrt{3}}{\sqrt{3}} = -3
 \end{aligned}$$

$$\begin{aligned}
 17.3. \quad & \cos\left(\frac{11\pi}{6}\right) + \sin\left(-\frac{5\pi}{3}\right) = \\
 & = \cos\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) - \sin\left(\frac{6\pi}{3} - \frac{\pi}{3}\right) = \\
 & = \cos\left(2\pi - \frac{\pi}{6}\right) - \sin\left(2\pi - \frac{\pi}{3}\right) = \\
 & = \cos\left(-\frac{\pi}{6}\right) - \sin\left(-\frac{\pi}{3}\right) = \\
 & = \cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right) = \\
 & = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 17.4. \quad & \sqrt{2}\sin\left(\frac{7\pi}{4}\right) + \sqrt{3}\tan\left(\frac{4\pi}{3}\right) = \\
 & = \sqrt{2}\sin\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) + \sqrt{3}\tan\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) = \\
 & = \sqrt{2}\sin\left(2\pi - \frac{\pi}{4}\right) + \sqrt{3}\tan\left(\pi + \frac{\pi}{3}\right) = \\
 & = \sqrt{2}\sin\left(-\frac{\pi}{4}\right) + \sqrt{3}\tan\frac{\pi}{3} = \\
 & = -\sqrt{2}\sin\frac{\pi}{4} + \sqrt{3} \times \sqrt{3} = \\
 & = -\sqrt{2} \times \frac{\sqrt{2}}{2} + 3 = -1 + 3 = 2
 \end{aligned}$$

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$$\begin{aligned}
 18.1. \quad & \tan\left(\frac{\pi}{2} - \theta\right) \times \tan \theta = \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos\left(\frac{\pi}{2} - \theta\right)} \times \frac{\sin \theta}{\cos \theta} = \\
 & = \frac{\cos \theta}{\sin \theta} \times \frac{\sin \theta}{\cos \theta} = 1
 \end{aligned}$$

$$\begin{aligned}
 18.2. \quad & \frac{\tan(\pi - \alpha) \times \sin\left(\frac{\pi}{2} + \alpha\right)}{\cos\left(\frac{3\pi}{2} - \alpha\right)} = \\
 & = \frac{\tan(-\alpha) \times \cos \alpha}{\cos\left[\pi + \left(\frac{\pi}{2} - \alpha\right)\right]} = \frac{-\tan \alpha \times \cos \alpha}{-\cos\left(\frac{\pi}{2} - \alpha\right)} = \\
 & = \frac{-\frac{\sin \alpha}{\cos \alpha} \times \cos \alpha}{-\sin \alpha} = \frac{\sin \alpha}{\sin \alpha} = 1
 \end{aligned}$$

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$$\begin{aligned}
 19.1. \quad & \sin(\pi - \alpha) + \cos\left(\frac{\pi}{2} - \alpha\right) + 2\sin(-\alpha) = \\
 & = \sin \alpha + \sin \alpha - 2\sin \alpha = \\
 & = 0
 \end{aligned}$$

$$\begin{aligned}
 19.2. \quad & \tan(-\alpha) \times \sin\left(\frac{\pi}{2} + \alpha\right) + \sin(\pi + \alpha) = \\
 & = -\tan \alpha \times \cos \alpha - \sin \alpha = \\
 & = -\frac{\sin \alpha}{\cos \alpha} \times \cos \alpha - \sin \alpha = \\
 & = -\sin \alpha - \sin \alpha = \\
 & = -2\sin \alpha
 \end{aligned}$$

$$\begin{aligned}
 19.3. \quad & \sin^2\left(\frac{\pi}{2} + \alpha\right) + \sin^2(5\pi - \alpha) = \\
 & = \left[\sin\left(\frac{\pi}{2} + \alpha\right)\right]^2 + [\sin(4\pi + \pi - \alpha)]^2 = \\
 & = \cos^2 \alpha + [\sin(\pi - \alpha)]^2 = \\
 & = \cos^2 \alpha + \sin^2 \alpha = 1
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \bullet \quad \cos\left(\frac{\pi}{2} + x\right) = \frac{12}{13} \wedge x \in \left]\pi, \frac{3\pi}{2}\right[\Leftrightarrow \\
 & \Leftrightarrow -\sin x = \frac{12}{13} \wedge x \in \left]\pi, \frac{3\pi}{2}\right[\Leftrightarrow \\
 & \Leftrightarrow \sin x = -\frac{12}{13} \wedge x \in \left]\pi, \frac{3\pi}{2}\right[\\
 & \bullet \quad 13\sin\left(x - \frac{\pi}{2}\right) - 5\tan(2\pi + x) = \\
 & = 13\sin\left[-\left(\frac{\pi}{2} - x\right)\right] - 5\tan x = \\
 & = -13\sin\left(\frac{\pi}{2} - x\right) - 5\tan x = \\
 & = -13\cos x - 5\tan x \\
 & \bullet \quad \sin^2 x + \cos^2 x = 1 \\
 & \left(-\frac{12}{13}\right)^2 + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \frac{144}{169} \Leftrightarrow \\
 & \Leftrightarrow \cos^2 x = \frac{25}{169} \Leftrightarrow \cos x = \pm \sqrt{\frac{25}{169}} \\
 & \text{Como } x \in 3.^\circ \text{ Q, então } \cos x = -\frac{5}{13}.
 \end{aligned}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

$$\begin{aligned} & -13\cos x - 5\tan x = \\ & = -13 \times \left(-\frac{5}{13}\right) - 5 \times \frac{12}{5} = 5 - 12 = -7 \end{aligned}$$

$$\begin{aligned} 21. \quad & 3\sin\left(x - \frac{\pi}{2}\right) - 2 = 0 \wedge x \in]0, \pi[\Leftrightarrow \\ & \Leftrightarrow \sin\left(x - \frac{\pi}{2}\right) = \frac{2}{3} \wedge x \in]0, \pi[\Leftrightarrow \\ & \Leftrightarrow -\sin\left(\frac{\pi}{2} - x\right) = \frac{2}{3} \wedge x \in]0, \pi[\Leftrightarrow \\ & \Leftrightarrow -\cos x = \frac{2}{3} \wedge x \in]0, \pi[\Leftrightarrow \\ & \Leftrightarrow \cos x = -\frac{2}{3} \wedge x \in \left]\frac{\pi}{2}, \pi\right[\end{aligned}$$

$$\begin{aligned} & \tan(\pi - x) + \cos\left(\frac{\pi}{2} + x\right) = \\ & = -\tan x - \sin x \end{aligned}$$

$$\begin{aligned} & \sin^2 x + \cos^2 x = 1 \\ & \sin^2 x + \left(-\frac{2}{3}\right)^2 = 1 \Leftrightarrow \sin^2 x = 1 - \frac{4}{9} \Leftrightarrow \\ & \Leftrightarrow \sin^2 x = \frac{5}{9} \Leftrightarrow \sin x = \pm \frac{\sqrt{5}}{3} \end{aligned}$$

$$\text{Como } x \in 2.^\circ \text{ Q, temos } \sin x = \frac{\sqrt{5}}{3}.$$

$$\tan x = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

$$\begin{aligned} & -\tan x - \sin x = \\ & = \frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{3} = \frac{3\sqrt{5} - 2\sqrt{5}}{6} = \frac{\sqrt{5}}{6} \end{aligned}$$

$$22. \text{ Sendo } \widehat{AOB} = \alpha, \widehat{AOC} = \frac{\pi}{2} + \alpha.$$

$$\text{A ordenada de } T \text{ é igual a } \tan\left(\frac{\pi}{2} + \alpha\right).$$

$$\text{Logo, } \tan\left(\frac{\pi}{2} + \alpha\right) = -2.$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -2 \Leftrightarrow \frac{\sin\left(\frac{\pi}{2} + \alpha\right)}{\cos\left(\frac{\pi}{2} + \alpha\right)} = -2 \Leftrightarrow$$

$$\Leftrightarrow \frac{\cos \alpha}{-\sin \alpha} = -2 \Leftrightarrow$$

$$\Leftrightarrow -\frac{1}{\tan \alpha} = -2 \Leftrightarrow$$

$$\Leftrightarrow \tan \alpha = \frac{1}{2}$$

No triângulo $[OAB]$, considerando $[OA]$ para base, temos que a altura é igual a $\sin \alpha$.

Logo, a área de $[OAB]$ é $A = \frac{\sin \alpha}{2}$ pois $\overline{OA} = 1$.

Determinemos $\sin \alpha$ sabendo que $\tan \alpha = \frac{1}{2}$.

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + \left(\frac{1}{2}\right)^2 = \frac{1}{\cos^2 \alpha} \Leftrightarrow \frac{1}{\cos^2 \alpha} = \frac{5}{4} \Leftrightarrow \cos^2 \alpha = \frac{4}{5}$$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\text{Como } \alpha \in 1.^\circ \text{ Q, } \sin \alpha = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

$$A_{[ABC]} = \frac{\sin \alpha}{2} = \frac{1}{2} \times \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{10}$$

Atividades complementares

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$$23. \quad \widehat{BC} = 2 \times 20^\circ = 40^\circ$$

$$\widehat{ADB} = 2 \times 130^\circ = 260^\circ$$

$$\widehat{AC} = 360^\circ - (260^\circ + 40^\circ) = 60^\circ$$

$$\widehat{AB} = 40^\circ + 60^\circ = 100^\circ$$

$$23.1. \text{ a) } \widehat{BOC} = 40^\circ$$

$$360^\circ - 40^\circ = 320^\circ$$

$$\alpha = 40^\circ \text{ ou } \alpha = -320^\circ$$

$$\text{b) } \widehat{BOA} = 100^\circ$$

$$360^\circ - 100^\circ = 260^\circ$$

$$\alpha = -100^\circ \text{ ou } \alpha = 260^\circ$$

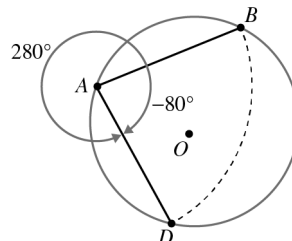
23.2. Se D é imagem de B numa rotação de centro A , então

$$\widehat{AD} = \widehat{AB}. \text{ Logo, } \widehat{AD} = \widehat{AB} = 100^\circ.$$

Assim, $\widehat{BD} = 360^\circ - (100^\circ + 100^\circ) = 160^\circ$ pelo que:

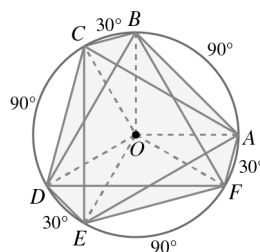
$$\widehat{DAB} = 160^\circ : 2 = 80^\circ$$

$$360^\circ - 80^\circ = 280^\circ$$



Portanto, $\beta = -80^\circ$ ou $\beta = 280^\circ$.

$$24.1. \text{ a) } \widehat{AOD} = 90^\circ + 30^\circ + 90^\circ = 210^\circ$$



Lado extremidade: $\widehat{OD}$

$$\text{b) } \widehat{AOE} = -30^\circ - 90^\circ = -120^\circ$$

Lado extremidade: $\widehat{OE}$

$$\text{c) } 330^\circ = 360^\circ - 30^\circ$$

$$\widehat{AOF} = 330^\circ$$

Lado extremidade: $\widehat{OF}$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

d) $\widehat{A\hat{O}D} = -30^\circ - 90^\circ - 30^\circ = -150^\circ$

Lado extremidade: $\hat{O}D$

24.2. a) $960^\circ = 240^\circ + 2 \times 360^\circ$

$960^\circ = (240^\circ, 2)$

$\widehat{A\hat{O}E} = 90^\circ + 30^\circ + 90^\circ + 30^\circ = 240^\circ$

Lado extremidade: $\hat{O}E$

b) $-1470^\circ = -30^\circ - 4 \times 360^\circ$

$-1470^\circ = (-30^\circ, 4)$

$\widehat{A\hat{O}F} = -30^\circ$

Lado extremidade: $\hat{O}F$

c) $1530^\circ = 90^\circ + 4 \times 360^\circ$

$1530^\circ = (90^\circ, 4)$

$\widehat{A\hat{O}B} = 90^\circ$

Lado extremidade: $\hat{O}B$

d) $-1320^\circ = -240^\circ - 3 \times 360^\circ$

$-1320^\circ = (-240^\circ, -3)$

$\widehat{A\hat{O}C} = -30^\circ - 90^\circ - 30^\circ - 90^\circ = -240^\circ$

Lado extremidade: $\hat{O}C$

e) $1200^\circ = 120^\circ + 3 \times 360^\circ$

$1200^\circ = (120^\circ, 3)$

$\widehat{A\hat{O}C} = 90^\circ + 30^\circ = 120^\circ$

Lado extremidade: $\hat{O}C$

f) $-990^\circ = -270^\circ - 2 \times 360^\circ$

$990^\circ = (-270^\circ, -2)$

$\widehat{A\hat{O}B} = -(360^\circ - 90^\circ) = -270^\circ$

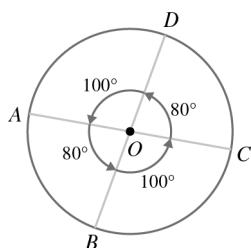
Lado extremidade: $\hat{O}B$

25. $\widehat{CD} = 40^\circ \times 2 = 80^\circ$; $\widehat{AB} = \widehat{CD} = 80^\circ$

$\widehat{AD} = 180^\circ - 80^\circ = 100^\circ$; $\widehat{BC} = \widehat{AD} = 100^\circ$

25.1. $\widehat{AB} = 80^\circ$, $\widehat{BC} = 100^\circ$, $\widehat{CD} = 80^\circ$ e $\widehat{DA} = 100^\circ$

25.2.



a) $540^\circ = 180^\circ + 360^\circ$

O transformado do ponto A é o ponto C.

b) $-640^\circ = -280^\circ - 360^\circ$

$\widehat{A\hat{O}B} = -280^\circ$

O transformado do ponto A é o ponto B.

c) $980^\circ = 260^\circ + 2 \times 360^\circ$

$\widehat{A\hat{O}D} = 260^\circ$

O transformado do ponto A é o ponto D.

d) $-900^\circ = -180^\circ - 2 \times 360^\circ$

O transformado do ponto A é o ponto C.

e) $1080^\circ = 0^\circ + 3 \times 360^\circ$

O transformado do ponto A é o ponto A.

$$\begin{array}{r} 960 \\ 240 \overline{) 360} \\ \underline{240} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

$$\begin{array}{r} 1470 \\ 30 \overline{) 360} \\ \underline{120} \\ 270 \\ \underline{270} \\ 0 \end{array}$$

$$\begin{array}{r} 1530 \\ 90 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

$$\begin{array}{r} 1320 \\ 240 \overline{) 360} \\ \underline{720} \\ 600 \\ \underline{600} \\ 0 \end{array}$$

$$\begin{array}{r} 1200 \\ 120 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

$$\begin{array}{r} 990 \\ 270 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

f) $-1180^\circ = -100^\circ - 3 \times 360^\circ$

$\widehat{A\hat{O}D} = -100^\circ$

O transformado do ponto A é o ponto D.

g) $1520^\circ = 80^\circ + 4 \times 360^\circ$

$\widehat{A\hat{O}B} = 80^\circ$

O transformado do ponto A é o ponto B.

h) $-1440^\circ = 0^\circ - 4 \times 360^\circ$

O transformado do ponto A é o ponto A.

$$\begin{array}{r} 1180 \\ 100 \overline{) 360} \\ \underline{360} \\ 820 \\ \underline{800} \\ 20 \end{array}$$

$$\begin{array}{r} 1520 \\ 80 \overline{) 360} \\ \underline{320} \\ 40 \end{array}$$

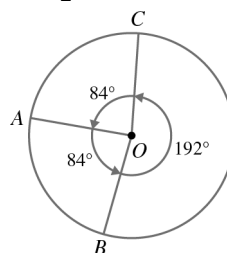
26.1. $360^\circ - 96^\circ = 264^\circ$

$\alpha = -96^\circ$ ou $\alpha = 264^\circ$

26.2. $96^\circ \times 2 = 192^\circ$

$360^\circ - 192^\circ = 168^\circ$

$\frac{180^\circ - 96^\circ}{2} = 42^\circ$; $42^\circ \times 2 = 84^\circ$

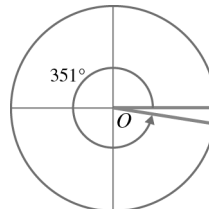


a) $\beta = -192^\circ$ ou $\beta = 168^\circ$

b) $\beta = 84^\circ$ ou $\beta = -276^\circ$

c) $\beta = -84^\circ$ ou $\beta = 276^\circ$

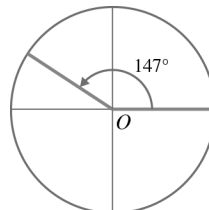
27.1. $351^\circ = 360^\circ - 9^\circ$



α pertence ao 4.º quadrante

$\sin \alpha < 0$ e $\cos \alpha > 0$

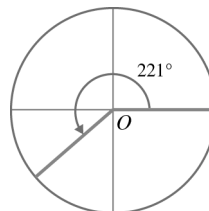
27.2. $147^\circ = 180^\circ - 33^\circ$



α pertence ao 2.º quadrante

$\sin \alpha > 0$ e $\cos \alpha < 0$

27.3. $221^\circ = 180^\circ + 41^\circ$



α pertence ao 3.º quadrante

$\sin \alpha < 0$ e $\cos \alpha < 0$

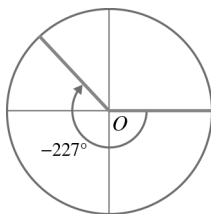
$$\begin{array}{r} 980 \\ 260 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

$$\begin{array}{r} 900 \\ 180 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

$$\begin{array}{r} 1080 \\ 0 \overline{) 360} \\ \underline{360} \\ 0 \end{array}$$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

27.4. $-227^\circ = -180^\circ - 47^\circ$



α pertence ao 2.º quadrante

$\sin \alpha > 0$ e $\cos \alpha < 0$

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28.1. $A(\cos 45^\circ, \sin 45^\circ)$ ou $A\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$M(1, \tan 45^\circ)$ ou $M(1, 1)$

C é a imagem de A na reflexão central de centro O .

Logo, $C\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

28.2. $P\hat{O}C = 180^\circ + 45^\circ = 225^\circ$. Logo, $C(\cos 225^\circ, \sin 225^\circ)$.

Temos, portanto, $\sin 225^\circ = -\frac{\sqrt{2}}{2}$ e $\cos 225^\circ = -\frac{\sqrt{2}}{2}$.

28.3. $P\hat{O}B = 180^\circ - 45^\circ = 135^\circ$; $B(\cos 135^\circ, \sin 135^\circ)$

$\cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

$\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

28.4. $P\hat{O}D = 360^\circ - 45^\circ = 315^\circ$; $D(\cos 315^\circ, \sin 315^\circ)$

Por outro lado, D é a imagem do ponto B pela reflexão

central de centro O . Assim, $D\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Logo, $\sin 315^\circ = -\frac{\sqrt{2}}{2}$ e $\cos 315^\circ = \frac{\sqrt{2}}{2}$.

28.5. N é a imagem de M pela reflexão de eixo Ox .

Como $M(1, 1)$, então N tem coordenadas $(1, -1)$.

29.1. $750^\circ = 30^\circ + 2 \times 360^\circ$

$750^\circ = (30^\circ, 2)$

$\frac{750}{30} \mid \frac{360}{2}$

$\sin 750^\circ = \sin 30^\circ = \frac{1}{2}$

$\cos 750^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 750^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$

29.2. $1140^\circ = 60^\circ + 3 \times 360^\circ$

$1140^\circ = (60^\circ, 3)$

$\frac{1140}{60} \mid \frac{360}{3}$

$\sin 1140^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 1140^\circ = \cos 60^\circ = \frac{1}{2}$

$\tan 1140^\circ = \tan 60^\circ = \sqrt{3}$

29.3. $405^\circ = 45^\circ + 360^\circ$

$405^\circ = (45^\circ, 1)$

$\sin 405^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 405^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 405^\circ = \tan 45^\circ = 1$

30.1. $480^\circ = 120^\circ + 360^\circ$; $480^\circ = (120^\circ, 1)$

$\sin 480^\circ = \sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 480^\circ = \cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$

30.2. $870^\circ = 150^\circ + 2 \times 360^\circ$

$870^\circ = (150^\circ, 2)$

$\frac{870}{150} \mid \frac{360}{2}$

$\sin 870^\circ = \sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$

$\cos 870^\circ = \cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

30.3. $1215^\circ = 135^\circ + 3 \times 360^\circ$

$1215^\circ = (135^\circ, 3)$

$\frac{1215}{135} \mid \frac{360}{3}$

$\sin 1215^\circ = \sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 1215^\circ = \cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

30.4. $810^\circ = 90^\circ + 2 \times 360^\circ$

$810^\circ = (90^\circ, 2)$

$\frac{810}{90} \mid \frac{360}{2}$

$\sin 810^\circ = \sin 90^\circ = 1$

$\cos 810^\circ = \cos 90^\circ = 0$

31.1.

Graus		Rad
180	—	π
240	—	x
$x = \frac{240\pi}{180} = \frac{4\pi}{3}$		
$240^\circ = \frac{4\pi}{3}$ rad		

31.2.

Graus		Rad
180	—	π
210	—	x
$x = \frac{210\pi}{180} = \frac{7\pi}{6}$		
$210^\circ = \frac{7\pi}{6}$ rad		

31.3.

Graus		Rad
180	—	π
300	—	x
$x = \frac{300\pi}{180} = \frac{5\pi}{3}$		
$300^\circ = \frac{5\pi}{3}$ rad		

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

31.4.

Graus		Rad
180	—	π
315	—	x
$x = \frac{315\pi}{180} = \frac{7\pi}{4}$		
$315^\circ = \frac{7\pi}{4} \text{ rad}$		

31.5.

Graus		Rad
180	—	π
-288	—	x
$x = \frac{-288\pi}{180} = -\frac{8\pi}{5}$		
$-288^\circ = -\frac{8\pi}{5} \text{ rad}$		

31.6.

Graus		Rad
180	—	π
420	—	x
$x = \frac{420\pi}{180} = \frac{7\pi}{3}$		
$420^\circ = \frac{7\pi}{3} \text{ rad}$		

32.1.

Graus		Rad
180	—	π
x	—	$\frac{7\pi}{5}$
$\frac{7\pi}{5} \text{ rad} = 252^\circ$		

32.2.

Graus		Rad
180	—	π
x	—	$-\frac{5\pi}{2}$
$x = \frac{-\frac{5\pi}{2} \times 180}{\pi} = -450, \text{ logo } -\frac{5\pi}{2} \text{ rad} = -450^\circ$		

32.3.

Graus		Rad
180	—	π
x	—	$\frac{3\pi}{8}$
$x = \frac{180 \times \frac{3\pi}{8}}{\pi} = 67,5, \text{ logo } \frac{3\pi}{8} \text{ rad} = 67,5^\circ$		

32.4.

Graus		Rad
180	—	π
x	—	$-\frac{5\pi}{16}$
$x = \frac{-\frac{5\pi}{16} \times 180}{\pi} = 56,25^\circ, \text{ logo } -\frac{5\pi}{16} \text{ rad} = -56,25^\circ$		

32.5.

Graus		Rad
180	—	π
x	—	$\frac{3\pi}{32}$
$x = \frac{180 \times \frac{3\pi}{32}}{\pi} = 16,875^\circ, \text{ logo } \frac{3\pi}{32} \text{ rad} = 16,875^\circ$		

32.6.

Graus		Rad
180	—	π
x	—	$-\frac{6\pi}{25}$
$x = \frac{-\frac{6\pi}{25} \times 180}{\pi} = -43,2^\circ, \text{ logo } -\frac{6\pi}{25} \text{ rad} = -43,2^\circ$		

33. $\widehat{ACB} = \pi - \frac{7\pi}{12} - \frac{\pi}{6} = \frac{12\pi - 7\pi - 2\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$

$\frac{\pi}{4} \text{ rad} = 45^\circ$

$\widehat{ACB} = \frac{\pi}{4} \text{ rad} = 45^\circ$

34. Se $[AC]$ é o lado de um triângulo equilátero inscrito na circunferência, então $\widehat{AC} = \frac{2\pi}{3} \text{ rad}$.

Como $\widehat{BC} = 2 \times \frac{2\pi}{15} = \frac{4\pi}{15} \text{ rad}$, temos que:

$$\widehat{AB} = \frac{2\pi}{3} - \frac{4\pi}{15} = \frac{10\pi - 4\pi}{15} = \frac{6\pi}{15} = \frac{2\pi}{5}$$

$$\frac{2\pi}{5} = 2\pi \times \frac{5}{2\pi} = 5$$

$[AB]$ é o lado de um pentágono regular inscrito na circunferência.

35.1. $T_1(1, \tan \alpha)$

$$\tan \alpha = 2$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + 2^2 = \frac{1}{\cos^2 \alpha} \Leftrightarrow \cos^2 \alpha = \frac{1}{5} \Leftrightarrow \cos \alpha = \pm \sqrt{\frac{1}{5}}$$

Como $\alpha \in 1.^\circ \text{ Q}$, $\cos \alpha = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$.

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \frac{1}{5} = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{1}{5} \Leftrightarrow \sin \alpha = \pm \sqrt{\frac{4}{5}}$$

Como $\alpha \in 1.^\circ \text{ Q}$, temos $\sin \alpha = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

$$\sin \alpha = \frac{2\sqrt{5}}{5}, \cos \alpha = \frac{\sqrt{5}}{5} \text{ e } \tan \alpha = 2$$

35.2. $P_2(\cos \beta, \sin \beta)$

$$\sin \beta = \frac{3}{4}$$

$$\sin^2 \beta + \cos^2 \beta = 1$$

$$\frac{9}{16} + \cos^2 \beta = 1 \Leftrightarrow \cos^2 \beta = 1 - \frac{9}{16} \Leftrightarrow \cos^2 \beta = \frac{7}{16}$$

Como $\beta \in 2.^\circ Q$, então $\cos \beta = -\frac{\sqrt{7}}{4}$.

$$\tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$$

$$\sin \beta = \frac{3}{4}, \cos \beta = -\frac{\sqrt{7}}{4} \text{ e } \tan \beta = -\frac{3\sqrt{7}}{4}$$

35.3. $P_3(\cos \theta, \sin \theta)$

$$\cos \theta = -\frac{7}{9}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \left(-\frac{7}{9}\right)^2 = 1 \Leftrightarrow \sin^2 \theta = 1 - \frac{49}{81} \Leftrightarrow \sin^2 \theta = \frac{32}{81}$$

Como $\theta \in 3.^\circ Q$, $\sin \theta = -\frac{\sqrt{32}}{9} = -\frac{4\sqrt{2}}{9}$.

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{4\sqrt{2}}{9}}{-\frac{7}{9}} = \frac{4\sqrt{2}}{7}$$

$$\sin \theta = -\frac{4\sqrt{2}}{9}, \cos \theta = -\frac{7}{9} \text{ e } \tan \theta = \frac{4\sqrt{2}}{7}$$

35.4. $T_4(1, \tan \varphi)$

$$\tan \varphi = -\frac{3}{4}$$

$$1 + \tan^2 \varphi = \frac{1}{\cos^2 \varphi}$$

$$1 + \left(-\frac{3}{4}\right)^2 = \frac{1}{\cos^2 \varphi} \Leftrightarrow 1 + \frac{9}{16} = \frac{1}{\cos^2 \varphi} \Leftrightarrow \frac{1}{\cos^2 \varphi} = \frac{25}{16} \Leftrightarrow \cos^2 \varphi = \frac{16}{25}$$

Como $\varphi \in 4.^\circ Q$, então $\cos \varphi = \frac{4}{5}$.

$$\sin^2 \varphi + \cos^2 \varphi = 1$$

$$\sin^2 \varphi + \left(\frac{4}{5}\right)^2 = 1 \Leftrightarrow \sin^2 \varphi = 1 - \frac{16}{25} \Leftrightarrow \sin^2 \varphi = \frac{9}{25}$$

Como $\varphi \in 4.^\circ Q$, temos $\sin \varphi = -\frac{3}{5}$.

$$\sin \varphi = -\frac{3}{5}, \cos \varphi = \frac{4}{5} \text{ e } \tan \varphi = -\frac{3}{4}$$

Como $0 < \hat{BDC} < \pi$ e $\tan(\hat{BDC}) > 0$, então $\cos(\hat{BDC}) > 0$.

$$\cos(\hat{BDC}) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\overline{BC}^2 = (\sqrt{5})^2 + 3^2 - 2\sqrt{5} \times 3 \times \frac{\sqrt{5}}{5} = 5 + 9 - 6 \times \frac{\sqrt{5} \times \sqrt{5}}{5} = 8$$

$$\overline{BC} = \sqrt{8} \text{ cm} = 2\sqrt{2} \text{ cm}$$

$$\begin{aligned} 37.1. \tan x + \frac{\cos x}{1 + \sin x} &= \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \\ &= \frac{\sin x(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)} = \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} = \\ &= \frac{\sin x + 1}{\cos x(\sin x + 1)} = \frac{1}{\cos x} \end{aligned}$$

$$\begin{aligned} 37.2. \sin^2 x - \sin^4 x &= \sin^2 x(1 - \sin^2 x) = \\ &= (1 - \cos^2 x) \times \cos^2 x = \cos^2 x - \cos^4 x \end{aligned}$$

$$\begin{aligned} 37.3. \frac{\sin x \cos x}{\sin x - \cos x} + \frac{\cos x}{1 - \tan x} &= \\ &= \frac{\sin x \cos x}{\sin x - \cos x} + \frac{\cos x}{1 - \frac{\sin x}{\cos x}} = \\ &= \frac{\sin x \cos x}{\sin x - \cos x} + \frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} = \\ &= \frac{\sin x \cos x}{\sin x - \cos x} - \frac{\cos^2 x}{\sin x - \cos x} = \\ &= \frac{\sin x \cos x - \cos^2 x}{\sin x - \cos x} = \frac{\cos x(\sin x - \cos x)}{\sin x - \cos x} = \cos x \end{aligned}$$

$$\begin{aligned} 37.4. \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} &= \\ &= \frac{1 + \sin x - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \\ &= \frac{1 + \sin x - 1 + \sin x}{1 - \sin^2 x} = \\ &= \frac{2\sin x}{\cos^2 x} = \frac{2\sin x}{\cos x} \times \frac{1}{\cos x} = \\ &= 2 \tan x \times \frac{1}{\cos x} = \frac{2 \tan x}{\cos x} \end{aligned}$$

$$\begin{aligned} 38.1. \left(\sin \frac{4\pi}{3} - \cos \frac{11\pi}{6}\right) \tan\left(-\frac{5\pi}{6}\right) &= \\ &= \left[\sin\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) - \cos\left(\frac{12\pi}{6} - \frac{\pi}{6}\right)\right] \times \left(-\tan \frac{5\pi}{6}\right) = \\ &= \left[-\sin\left(\pi + \frac{\pi}{3}\right) - \cos\left(2\pi - \frac{\pi}{6}\right)\right] \times \tan\left(\frac{6\pi}{6} - \frac{\pi}{6}\right) = \\ &= -\left(-\sin \frac{\pi}{3} - \cos\left(-\frac{\pi}{6}\right)\right) \times \tan\left(\pi - \frac{\pi}{6}\right) = \\ &= -\left(-\frac{\sqrt{3}}{2} - \cos \frac{\pi}{6}\right) \times \tan\left(-\frac{\pi}{6}\right) = \\ &= -\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) \times \left(-\tan \frac{\pi}{6}\right) = \\ &= -(-\sqrt{3}) \times \left(-\frac{\sqrt{3}}{3}\right) = -\frac{\sqrt{3} \times \sqrt{3}}{3} = -\frac{3}{3} = -1 \end{aligned}$$

36. $\overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2 \Leftrightarrow \overline{BD}^2 = 2^2 + 1^2$

$$\overline{BD} = \sqrt{5} \text{ cm}$$

$$\overline{BC}^2 = \overline{BD}^2 + \overline{DC}^2 - 2\overline{BD} \times \overline{DC} \times \cos(\hat{BDC})$$

Determinemos $\cos(\hat{BDC})$, sabendo que $\tan(\hat{BDC}) = 2$.

$$1 + \tan^2(\hat{BDC}) = \frac{1}{\cos^2(\hat{BDC})}$$

$$1 + 2^2 = \frac{1}{\cos^2(\hat{BDC})} \Leftrightarrow \cos^2 \hat{BDC} = \frac{1}{5}$$

$$\begin{aligned}
 38.2. \quad & \sqrt{2} \sin \frac{7\pi}{4} + \tan \frac{9\pi}{4} + \cos \frac{5\pi}{2} = \\
 & = \sqrt{2} \sin \left(\frac{8\pi}{4} - \frac{\pi}{4} \right) + \tan \left(\frac{8\pi}{4} + \frac{\pi}{4} \right) + \cos \left(\frac{4\pi}{2} + \frac{\pi}{2} \right) = \\
 & = \sqrt{2} \sin \left(2\pi - \frac{\pi}{4} \right) + \tan \left(2\pi + \frac{\pi}{4} \right) + \cos \left(2\pi + \frac{\pi}{2} \right) = \\
 & = \sqrt{2} \sin \left(-\frac{\pi}{4} \right) + \tan \frac{\pi}{4} + \cos \frac{\pi}{2} = \\
 & = -\sqrt{2} \times \frac{\sqrt{2}}{2} + 1 = \\
 & = -\frac{2}{2} + 1 = \\
 & = -1 + 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 38.3. \quad & \frac{\cos \frac{11\pi}{4} \times \sin \left(-\frac{7\pi}{6} \right)}{\sqrt{2} \sin^2 \frac{\pi}{5} + \sqrt{2} \cos^2 \frac{\pi}{5}} = \\
 & = \frac{-\cos \left(\frac{8\pi}{4} + \frac{3\pi}{4} \right) \sin \left(\frac{6\pi}{6} + \frac{\pi}{6} \right)}{\sqrt{2} \left(\sin^2 \frac{\pi}{5} + \cos^2 \frac{\pi}{5} \right)} = \\
 & = \frac{-\cos \left(2\pi + \frac{3\pi}{4} \right) \times \sin \left(\pi + \frac{\pi}{6} \right)}{\sqrt{2} \times 1} = \\
 & = \frac{-\cos \left(\frac{4\pi}{4} - \frac{\pi}{4} \right) \times \left(-\sin \frac{\pi}{6} \right)}{\sqrt{2}} = \\
 & = \frac{-\cos \left(\pi - \frac{\pi}{4} \right) \times \sin \frac{\pi}{6}}{\sqrt{2}} = \\
 & = \frac{-\cos \frac{\pi}{4} \times \frac{1}{2}}{\sqrt{2}} = \frac{-\frac{\sqrt{2}}{2} \times \frac{1}{2}}{\sqrt{2}} = \\
 & = \frac{-\sqrt{2} \times \frac{1}{2} \times \frac{1}{2}}{\sqrt{2}} = -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 38.4. \quad & \frac{2 \cos \frac{11\pi}{3} + \tan \frac{4\pi}{3}}{3 \tan \left(-\frac{11\pi}{6} \right) + 2 \sin \left(\frac{11\pi}{6} \right)} = \\
 & = \frac{2 \cos \left(\frac{12\pi}{3} - \frac{\pi}{3} \right) + \tan \left(\frac{3\pi}{3} + \frac{\pi}{3} \right)}{-3 \tan \frac{11\pi}{6} + 2 \sin \left(\frac{12\pi}{6} - \frac{\pi}{6} \right)} = \\
 & = \frac{2 \cos \left(4\pi - \frac{\pi}{3} \right) + \tan \left(\pi + \frac{\pi}{3} \right)}{-3 \tan \left(\frac{12\pi}{6} - \frac{\pi}{6} \right) + 2 \sin \left(2\pi - \frac{\pi}{6} \right)} = \\
 & = \frac{2 \cos \left(-\frac{\pi}{3} \right) + \tan \frac{\pi}{3}}{-3 \tan \left(2\pi - \frac{\pi}{6} \right) + 2 \sin \left(-\frac{\pi}{6} \right)} = \\
 & = \frac{2 \cos \frac{\pi}{3} + \sqrt{3}}{-3 \tan \left(-\frac{\pi}{6} \right) - 2 \sin \frac{\pi}{6}} =
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{2 \times \frac{1}{2} + \sqrt{3}}{3 \tan \frac{\pi}{6} - 2 \times \frac{1}{2}} = \frac{1 + \sqrt{3}}{3 \times \frac{\sqrt{3}}{3} - 1} = \\
 & = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} - 1)} = \frac{3 + 2\sqrt{3} + 1}{3 - 1} = \\
 & = \sqrt{3} + 2
 \end{aligned}$$

$$\begin{aligned}
 39.1. \quad & \left[\sin \left(\frac{\pi}{2} + x \right) - \sin (x - \pi) \right]^2 - 1 = \\
 & = (\cos x + \sin (\pi - x))^2 - 1 = \\
 & = (\cos x + \sin x)^2 - 1 = \\
 & = \cos^2 x + 2 \sin x \cos x + \sin^2 x - 1 = \\
 & = (\sin^2 x + \cos^2 x) + 2 \sin x \cos x - 1 = \\
 & = 1 + 2 \sin x \cos x - 1 = 2 \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 39.2. \quad & \cos \left(\frac{5\pi}{2} + x \right) \times \sin (\pi + x) \times \tan \left(\frac{\pi}{2} - x \right) = \\
 & = \cos \left(\frac{4\pi}{2} + \frac{\pi}{2} + x \right) \times (-\sin x) \times \frac{\sin \left(\frac{\pi}{2} - x \right)}{\cos \left(\frac{\pi}{2} - x \right)} = \\
 & = -\cos \left(2\pi + \frac{\pi}{2} + x \right) \times \sin x \times \frac{\cos x}{\sin x} = \\
 & = -\cos \left(\frac{\pi}{2} + x \right) \times \cos x = \\
 & = -(-\sin x) \times \cos x = \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 40.1. \quad & A(1, 0), P(\cos x, \sin x), Q(1, \tan x) \\
 & \text{Área de } [AQP] = \text{Área de } [OAQ] - \text{Área } [OAP] = \\
 & = \frac{\overline{OA} \times \overline{AQ}}{2} - \frac{\overline{OA} \times \text{ordenada de } P}{2} = \\
 & = \frac{1 \times \tan x}{2} - \frac{1 \times \sin x}{2} = \frac{\tan x - \sin x}{2}
 \end{aligned}$$

$$\begin{aligned}
 40.2. \quad & \text{Como } \overline{OP} = \overline{OA} = 1, \text{ se } \overline{AP} = \overline{OP} \text{ então} \\
 & \overline{OA} = \overline{AP} = \overline{OP} = 1, \text{ ou seja, o triângulo } [OAP] \text{ é equilátero,} \\
 & \text{pelo que } x = \frac{\pi}{3}.
 \end{aligned}$$

$$\begin{aligned}
 \text{a) } A_{[AQP]} &= \frac{\tan \frac{\pi}{3} - \sin \frac{\pi}{3}}{2} = \frac{1}{2} \left(\sqrt{3} - \frac{\sqrt{3}}{2} \right) = \\
 &= \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \quad & \overline{OA} = 1, \overline{AP} = 1 \text{ e } \overline{AQ} = \tan \frac{\pi}{3} = \sqrt{3} \\
 & \overline{OQ}^2 = \overline{OA}^2 + \overline{AQ}^2 \\
 & \overline{OQ}^2 = 1^2 + (\sqrt{3})^2 \Leftrightarrow \overline{OQ} = \sqrt{1+3} \Leftrightarrow \overline{OQ} = 2 \\
 & \overline{PQ} = \overline{OQ} - \overline{OP} = 2 - 1 = 1 \\
 & \text{Perímetro de } [AQP] = \overline{AQ} + \overline{PQ} + \overline{AP} = \\
 & = \sqrt{3} + 1 + 1 = \sqrt{3} + 2
 \end{aligned}$$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

$$40.3. \sin\left(\frac{\pi}{2} + x\right) - \cos(\pi - x) = \frac{8}{5} \Leftrightarrow$$

$$\Leftrightarrow \cos x + \cos x = \frac{8}{5} \Leftrightarrow 2\cos x = \frac{8}{5} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{4}{5}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x + \left(\frac{4}{5}\right)^2 = 1 \Leftrightarrow \sin^2 x = 1 - \frac{16}{25} \Leftrightarrow \sin^2 x = \frac{9}{25}$$

$$\text{Como } x \in 1.^\circ \text{ Q, então } \sin x = \frac{3}{5}.$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$A_{[AQP]} = \frac{\frac{3}{4} \cdot \frac{3}{5}}{2} = \frac{\frac{9}{20}}{2} = \frac{9}{40}$$

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$$41.1. \sin \frac{11\pi}{12} = \sin\left(\frac{12\pi}{12} - \frac{\pi}{12}\right) = \sin\left(\pi - \frac{\pi}{12}\right) =$$

$$= \sin \frac{\pi}{12} = a$$

$$41.2. \sin \frac{13\pi}{12} = \sin\left(\frac{12\pi}{12} + \frac{\pi}{12}\right) = \sin\left(\pi + \frac{\pi}{12}\right) =$$

$$= -\sin \frac{\pi}{12} = -a$$

$$41.3. \cos \frac{5\pi}{12} = \cos\left(\frac{6\pi}{12} - \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right) =$$

$$= \sin \frac{\pi}{12} = a$$

$$41.4. \cos \frac{7\pi}{12} = \cos\left(\frac{6\pi}{12} + \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{2} + \frac{\pi}{12}\right) =$$

$$= -\sin \frac{\pi}{12} = -a$$

$$42.1. A_{[PQB]} = A_{[OQB]} - A_{[OPB]} =$$

$$= \frac{\overline{OB} \times \overline{OA}}{2} - \frac{\overline{OB} \times \text{abscissa de } P}{2} =$$

$$= \frac{1 \times 1}{2} - \frac{1 \times \cos x}{2} =$$

$$= \frac{1}{2} - \frac{\cos x}{2} = \frac{1 - \cos x}{2}$$

$$42.2. 9\sin\left(\pi + x\right) + 8\cos\left(\frac{\pi}{2} + x\right) + 15 = 0 \Leftrightarrow$$

$$\Leftrightarrow -9\sin x - 8\sin x = -15 \Leftrightarrow$$

$$\Leftrightarrow -17\sin x = -15 \Leftrightarrow \sin x = \frac{15}{17}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{15}{17}\right)^2 + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \frac{225}{289} \Leftrightarrow \cos^2 x = \frac{64}{289}$$

$$\text{Como } x \in \left]0, \frac{\pi}{2}\right[, \text{ então } \cos x = \sqrt{\frac{64}{289}} = \frac{8}{17}.$$

$$A_{[PQB]} = \frac{1 - \frac{8}{17}}{2} = \frac{\frac{9}{17}}{2} = \frac{9}{34}$$

$$42.3. \text{ Se } x = \frac{\pi}{3} :$$

$$P\left(\cos \frac{\pi}{3}, \sin \frac{\pi}{3}\right) \text{ ou } P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$Q\left(1, \tan \frac{\pi}{3}\right) \text{ ou } Q(1, \sqrt{3})$$

$$\overline{PQ} = \sqrt{\left(1 - \frac{1}{2}\right)^2 + \left(\sqrt{3} - \frac{\sqrt{3}}{2}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$$

$$43.1. 810^\circ = 90^\circ + 2 \times 360^\circ$$

$$\begin{array}{r|l} 810 & 360 \\ \hline 90 & 2 \end{array}$$

O ponteiro dos minutos deu 2 voltas + $\frac{1}{4}$ de

volta. Logo, passaram 2 h 15 min após as 4 horas.

O relógio marca 6 h 15 min.

$$43.2. 360^\circ : 12 = 30$$

Em cada hora o ponteiro das horas descreve um ângulo de 30° de amplitude.

$$60 \text{ min} \quad \text{---} \quad 30^\circ$$

$$15 \text{ min} \quad \text{---} \quad x$$

$$x = \frac{15 \times 30}{60} = 7,5; \quad 90^\circ + 7,5^\circ = 97,5^\circ$$

$$\begin{array}{cc} \text{Graus} & \text{Rad} \\ 180 & \text{---} \quad \pi \\ 97,5 & \text{---} \quad x \end{array}$$

$$x = \frac{97,5\pi}{180} = \frac{13\pi}{24}$$

Às 6 h 15 min os ponteiros formam um ângulo de $\frac{13\pi}{24}$ rad de amplitude.



$$44.1. \cos^2 \frac{5\pi}{11} + \cos^2 \frac{\pi}{22} + \cos^2 \frac{7\pi}{12} + \cos^2 \frac{\pi}{12} =$$

$$= \cos^2\left(\frac{10\pi}{22}\right) + \cos^2 \frac{\pi}{22} + \cos^2\left(\frac{6\pi}{12} + \frac{\pi}{12}\right) + \cos^2 \frac{\pi}{12}$$

$$= \cos^2\left(\frac{11\pi}{22} - \frac{\pi}{22}\right) + \cos^2 \frac{\pi}{22} + \left[\cos\left(\frac{\pi}{2} + \frac{\pi}{12}\right)\right]^2 + \cos^2 \frac{\pi}{12}$$

$$= \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right)\right]^2 + \cos^2 \frac{\pi}{22} + \left(-\sin \frac{\pi}{12}\right)^2 + \cos^2 \frac{\pi}{12}$$

$$= \left(\sin^2 \frac{\pi}{22} + \cos^2 \frac{\pi}{22}\right) + \left(\sin^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{12}\right) = 1 + 1 = 2$$

$$44.2. \sin \frac{4\pi}{9} + \sin \frac{5\pi}{9} + \sin \frac{14\pi}{9} + \sin \frac{13\pi}{9} =$$

$$= \sin \frac{4\pi}{9} + \sin \frac{5\pi}{9} + \sin\left(\frac{9\pi}{9} + \frac{5\pi}{9}\right) + \sin\left(\frac{9\pi}{9} + \frac{4\pi}{9}\right) =$$

$$= \sin \frac{4\pi}{9} + \sin \frac{5\pi}{9} + \sin\left(\pi + \frac{5\pi}{9}\right) + \sin\left(\pi + \frac{4\pi}{9}\right) =$$

$$= \sin \frac{4\pi}{9} + \sin \frac{5\pi}{9} - \sin \frac{5\pi}{9} - \sin \frac{4\pi}{9} = 0$$

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

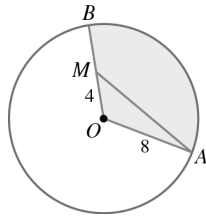
$$\begin{aligned}
 44.3. \quad & \sin \frac{\pi}{13} + \cos \frac{11\pi}{25} + \cos \frac{14\pi}{25} + \cos \frac{15\pi}{26} = \\
 & = \sin \frac{\pi}{13} + \cos \frac{11\pi}{25} + \cos \left(\frac{25\pi}{25} - \frac{11\pi}{25} \right) + \cos \left(\frac{13\pi}{26} + \frac{2\pi}{26} \right) = \\
 & = \sin \frac{\pi}{13} + \cos \frac{11\pi}{25} + \cos \left(\pi - \frac{11\pi}{25} \right) + \cos \left(\frac{\pi}{2} + \frac{\pi}{13} \right) = \\
 & = \sin \frac{\pi}{13} + \cos \frac{11\pi}{25} - \cos \frac{11\pi}{25} - \sin \frac{\pi}{13} = 0
 \end{aligned}$$

45.1.

Amplitude	Área
2π	$\pi \times 8^2$
x	$\frac{64\pi}{3}$

$$x = \frac{2\pi \times \frac{64}{3}\pi}{64\pi} = \frac{2\pi}{3}, \text{ logo } \widehat{AOB} = \frac{2\pi}{3} \text{ rad.}$$

$$\begin{aligned}
 45.2. \quad \overline{MA}^2 &= 8^2 + 4^2 - 2 \times 8 \times 4 \times \cos \frac{2\pi}{3} = \\
 &= 64 + 16 - 2 \times 32 \cos \left(\pi - \frac{\pi}{3} \right) \\
 &= 80 + 2 \times 32 \cos \frac{\pi}{3} \\
 &= 80 + 2 \times 32 \times \frac{1}{2} = 112 \\
 \overline{MA} &= \sqrt{112} \text{ cm} = 4\sqrt{7} \text{ cm}
 \end{aligned}$$



Avaliação 2

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1. Se $x \in \left] \pi, \frac{3\pi}{2} \right]$, $\sin x < 0$, $\cos x < 0$ e $\tan x > 0$.

- $\sin x \times \tan(-x) = -\sin x \times \tan x > 0$
- $\cos(-x) + \sin(\pi - x) = \cos x + \sin x < 0$
- $\sin(-x) \times \cos x = -\sin x \times \cos x < 0$
- $\sin\left(\frac{\pi}{2} - x\right) \times \tan x = \cos x \times \tan x < 0$

Resposta: (A)

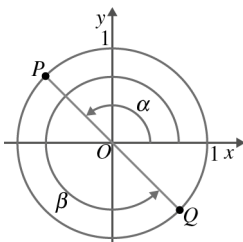
2. $d = \frac{4\pi}{5}$

$$a = c = \pi - \frac{4\pi}{5} = \frac{\pi}{5}$$

$$a + c = \frac{\pi}{5} + \frac{\pi}{5} = \frac{2\pi}{5}$$

Resposta: (A)

3. $P(x, y)$ e $Q(-x, -y)$



P e Q têm coordenadas simétricas

$$(A) \quad -\frac{7\pi}{6} = -\frac{6\pi}{6} - \frac{\pi}{6} = -\pi - \frac{\pi}{6}$$

P é do 2.º quadrante.

$$\frac{5\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \pi - \frac{\pi}{6}$$

Q é do 2.º quadrante.

[PQ] não é um diâmetro da circunferência.

$$(B) \quad -\frac{11\pi}{6} = -\frac{12\pi}{6} + \frac{\pi}{6} = -2\pi + \frac{\pi}{6}$$

P é do 1.º quadrante

$$\begin{aligned}
 \frac{19\pi}{6} &= \frac{12\pi}{6} + \frac{7\pi}{6} = 2\pi + \frac{6\pi}{6} + \frac{\pi}{6} = \\
 &= 2\pi + \pi + \frac{\pi}{6}
 \end{aligned}$$

Q é do 3.º quadrante

$$\cos\left(-\frac{11\pi}{6}\right) = \cos\left(-2\pi + \frac{\pi}{6}\right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{11\pi}{6}\right) = \sin\left(-2\pi + \frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\cos\left(\frac{19\pi}{6}\right) = \cos\left(2\pi + \pi + \frac{\pi}{6}\right) =$$

$$= \cos\left(\pi + \frac{\pi}{6}\right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{19\pi}{6}\right) = \sin\left(2\pi + \pi + \frac{\pi}{6}\right) =$$

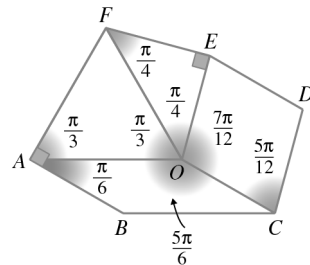
$$= \sin\left(\pi + \frac{\pi}{6}\right) = -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$Q\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

Logo, [PQ] é um diâmetro da circunferência.

Resposta: (B)

4.



$$\widehat{FAO} = \widehat{FOA} = \frac{\pi}{3}; \widehat{OFE} = \widehat{EOF} = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

$$\widehat{BAO} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}; \widehat{AOC} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\widehat{COE} = 2\pi - \frac{\pi}{4} - \frac{\pi}{3} - \frac{5\pi}{6} = \frac{7\pi}{12}; \widehat{DCO} = \pi - \frac{7\pi}{12} = \frac{5\pi}{12}$$

Resposta: (A)

1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

5. O ponto T tem coordenadas $(1, \tan \theta)$ sendo θ o ângulo cujo lado origem é o semieixo positivo Ox e o lado extremidade é a semirreta $\overrightarrow{OT}$.

Logo, $\tan \theta = -\frac{\sqrt{3}}{3}$, pelo que $\theta = -\frac{\pi}{6}$ e o ponto P tem

coordenadas $\left(\cos\left(\pi - \frac{\pi}{6}\right), \sin\left(\pi - \frac{\pi}{6}\right)\right)$.

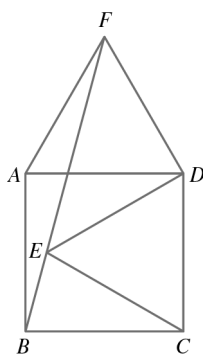
$$\cos\left(\pi - \frac{\pi}{6}\right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\pi - \frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}$$

Logo, $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

Resposta: (D)

6.



6.1. $\widehat{ADE} = \widehat{ADC} - \widehat{EDC} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

6.2. $\widehat{FDE} = \widehat{FDA} + \widehat{ADE} = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

Logo, o triângulo $[DFE]$ é retângulo em D .

Como $\overline{FD} = \overline{AD}$ e $\overline{ED} = \overline{DC} = \overline{AD}$ vem $\overline{FD} = \overline{AD}$. Assim, o triângulo $[DFE]$ é isósceles.

6.3. $\widehat{DEF} = \frac{\pi}{4}$ (o triângulo $[DEF]$ é retângulo em D e isósceles)

$\widehat{CED} = \frac{\pi}{3}$ (o triângulo $[CDE]$ é equilátero)

$\widehat{ECB} = \widehat{DCB} - \widehat{DCE} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$\widehat{CBE} = \widehat{BEC}$ porque o triângulo $[BCE]$ é isósceles, sendo $\overline{EC} = \overline{BC}$.

Logo, $\widehat{BEC} = \frac{1}{2}(\pi - \widehat{ECB}) = \frac{1}{2}\left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{12}$

$\widehat{BEF} = \widehat{DEF} + \widehat{CED} + \widehat{BEC} = \frac{\pi}{4} + \frac{\pi}{3} + \frac{5\pi}{12} = \pi$

Como o ângulo BEF é raso, o ponto E pertence à reta BF .

7. $360^\circ : 20 = 18^\circ$

- 7.1. Para obter 90 pontos a roda terá de rodar oito setores, para além das voltas inteiras.

$8 \times 18^\circ = 144^\circ$

$144^\circ + 360^\circ = 504^\circ$

$144 + 2 \times 360 = 864^\circ$

A amplitude do arco descrito por P_0 pode ser 504° ou 864° , por exemplo.

7.2. $810^\circ = 90^\circ + 2 \times 360$

$90^\circ : 18^\circ = 5$

O 1.º jogador obteve 60 pontos (5 setores)

$\frac{16\pi}{5} \text{ rad} = \frac{16}{5} \times 180^\circ = 576^\circ$

$576^\circ = 216^\circ + 360^\circ$

$216^\circ : 18 = 12$

O 2.º jogador obteve 25 pontos (12 setores)

$7\pi \text{ rad} = 7 \times 180^\circ = 1260^\circ$

$1260^\circ : 18 = 70$

O 3.º jogador obteve 5 pontos (10 setores)

A melhor pontuação (60 pontos) foi obtida pelo 1.º jogador.

8. $360^\circ : 12 = 30^\circ$

Em cada hora, o ponteiro das horas descreve um arco de 30° de amplitude.

$60 \text{ min} \longrightarrow 30^\circ$

$20 \text{ min} \longrightarrow x$

$x = \frac{20 \times 30}{60} = 10$

$30^\circ - 10^\circ = 20^\circ$

$20^\circ + 4 \times 30^\circ = 140^\circ$

Graus Rad

$180 \longrightarrow \pi$

$140 \longrightarrow x$

$x = \frac{140\pi}{180} = \frac{7\pi}{9}$

Às 11 h 20 min, os ponteiros formam um ângulo de amplitude $\frac{7\pi}{9} \text{ rad}$.



9.1. $\sin\left(-\frac{2\pi}{3}\right) - \cos\frac{17\pi}{6} - \tan\left(-\frac{5\pi}{4}\right) =$
 $= -\sin\frac{2\pi}{3} - \cos\left(\frac{12\pi}{6} + \frac{5\pi}{6}\right) + \tan\left(\frac{5\pi}{4}\right) =$
 $= -\sin\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) - \cos\left(2\pi + \frac{5\pi}{6}\right) + \tan\left(\frac{4\pi}{4} + \frac{\pi}{4}\right) =$
 $= -\sin\left(\pi - \frac{\pi}{3}\right) - \cos\left(\pi - \frac{\pi}{6}\right) + \tan\left(\pi + \frac{\pi}{4}\right) =$
 $= -\sin\frac{\pi}{3} + \cos\frac{\pi}{6} + \tan\frac{\pi}{4} = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 1 = 1$

9.2. $\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) - \tan^2\left(\frac{7\pi}{4}\right) =$
 $= \sin^2\left(\frac{\pi}{8}\right) + \left[\sin\left(\frac{4\pi}{8} - \frac{\pi}{8}\right)\right]^2 - \left[\tan\left(\frac{8\pi}{4} - \frac{\pi}{4}\right)\right]^2 =$
 $= \sin^2\left(\frac{\pi}{8}\right) + \left[\sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right]^2 - \left[\tan\left(2\pi - \frac{\pi}{4}\right)\right]^2 =$
 $= \sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right) - \left[\tan\left(-\frac{\pi}{4}\right)\right]^2 = 1 - (-1)^2 = 0$

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1.2. Ângulos generalizados. Fórmulas trigonométricas. Redução ao primeiro quadrante

$$\begin{aligned}
 10. \quad 1 + \cos\left(x - \pi\right) \times \sin\left(x - \frac{3\pi}{2}\right) &= \\
 &= 1 - \cos x \times \left[-\sin\left(\frac{3\pi}{2} - x\right)\right] = \\
 &= 1 + \cos x \times \sin\left[\pi + \left(\frac{\pi}{2} - x\right)\right] = \\
 &= 1 - \cos x \times \sin\left(\frac{\pi}{2} - x\right) = \\
 &= 1 - \cos x \times \cos x = \\
 &= 1 - \cos^2 x = \\
 &= \sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \tan^2 x - \sin^2 x &= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \\
 &= \sin^2 x \times \left(\frac{1}{\cos^2 x} - 1\right) = \\
 &= \sin^2 x \times \frac{1 - \cos^2 x}{\cos^2 x} = \\
 &= \sin^2 x \times \frac{\sin^2 x}{\cos^2 x} = \\
 &= \sin^2 x \times \tan^2 x = \\
 &= \tan^2 x \times \sin^2 x
 \end{aligned}$$

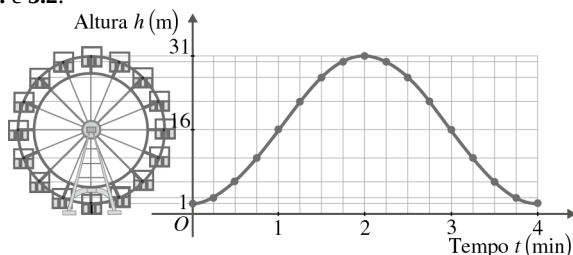
$$\begin{aligned}
 12.1. \quad \text{Base do triângulo } [PQR] &= \\
 &= \overline{RQ} = \overline{OA} = 1 \\
 \text{Altura do triângulo } [PQR] &= \\
 &= \text{ordenada de } Q - \text{ordenada de } P = \\
 &= \tan x - \sin x \\
 \text{Logo, } A_{[PQR]} &= \frac{\tan x - \sin x}{2}.
 \end{aligned}$$

$$\begin{aligned}
 12.2. \quad \text{Abcissa do ponto } P &= \cos x \\
 \cos x &= \frac{3}{5} \\
 \sin^2 x + \cos^2 x &= 1 \\
 \sin^2 x + \left(\frac{3}{5}\right)^2 &= 1 \Leftrightarrow \\
 \Leftrightarrow \sin^2 x &= 1 - \frac{9}{25} \Leftrightarrow \\
 \Leftrightarrow \sin^2 x &= \frac{16}{25} \\
 \text{Como } x \in \left]0, \frac{\pi}{2}\right[, \sin x &= \frac{4}{5}. \\
 \tan x &= \frac{\sin x}{\cos x} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3} \\
 A_{[PQR]} &= \frac{\frac{4}{3} - \frac{4}{5}}{2} = \frac{\frac{8}{15}}{2} = \frac{4}{15}
 \end{aligned}$$

Atividade inicial 3

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3.1. e 3.2.



3.3. A altura máxima foi de 31 metros e em cada volta ocorre uma vez.

3.4. A cabine esteve a 16 metros de altura após 1 minuto e após 3 minutos do instante inicial. Em cada volta esta altura ocorre duas vezes.

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1. $f(x) = 3 - 2\sin\left(\frac{x}{2}\right); D_f = \mathbb{R}$

1.1. $x \in \mathbb{R} \Leftrightarrow \frac{x}{2} \in \mathbb{R} \Leftrightarrow$

$$\Leftrightarrow -1 \leq \sin\left(\frac{x}{2}\right) \leq 1 \Leftrightarrow$$

$$\Leftrightarrow 2 \geq -2\sin\left(\frac{x}{2}\right) \geq -2 \Leftrightarrow$$

$$\Leftrightarrow -2 \leq -2\sin\left(\frac{x}{2}\right) \leq 2 \Leftrightarrow$$

$$\Leftrightarrow 3 - 2 \leq 3 - 2\sin\left(\frac{x}{2}\right) \leq 3 + 2 \Leftrightarrow$$

$$\Leftrightarrow 1 \leq f(x) \leq 5, \text{ logo } D'_f = [1, 5].$$

1.2. $0 \notin D'_f$. Logo, a função f não tem zeros.

1.3. Se $x \in D_f$, $x + 4\pi \in D_f$ porque $D_f = \mathbb{R}$

$$\begin{aligned} f(x + 4\pi) &= 3 - 2\sin\left(\frac{x + 4\pi}{2}\right) = 3 - 2\sin\left(\frac{x}{2} + \frac{4\pi}{2}\right) = \\ &= 3 - 2\sin\left(\frac{x}{2} + 2\pi\right) = 3 - 2\sin\frac{x}{2} = \\ &= f(x) \end{aligned}$$

$$\forall x \in D_f, f(x + 4\pi) = f(x)$$

Portanto, a função f é periódica de período 4π .

1.4. $f(a) + f(-a) = 3 - 2\sin\frac{a}{2} + 3 - 2\sin\left(\frac{-a}{2}\right) =$

$$= 6 - 2\sin\frac{a}{2} - 2\sin\left(-\frac{a}{2}\right) =$$

$$= 6 - \sin\frac{a}{2} + 2\sin\frac{a}{2} = \quad (\sin(-x) = -\sin x, \forall x \in \mathbb{R})$$

$$= 6$$

2. $g(x) = 1 + 2\sin\left(x - \frac{\pi}{2}\right)$

O gráfico da função g pode ser obtido do gráfico de $y = \sin x$ por uma dilatação vertical de coeficiente 2 seguida de uma translação de vetor $\left(\frac{\pi}{2}, 1\right)$.

3.1. $\arcsin\frac{\sqrt{2}}{2} = x \Leftrightarrow \frac{\sqrt{2}}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$$\Leftrightarrow x = \frac{\pi}{4}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = y \Leftrightarrow -\frac{\sqrt{3}}{2} = \sin y \wedge y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$$

$$\Leftrightarrow y = -\frac{\pi}{3}$$

$$\arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{3\pi + 4\pi}{12} =$$

$$= \frac{7\pi}{12}$$

3.2. $\arcsin\left(\frac{1}{2} + \sin\frac{\pi}{6}\right) = \arcsin\left(\frac{1}{2} + \frac{1}{2}\right) = \arcsin(1) = \frac{\pi}{2}$

3.3. $2\arcsin\left(\sin\frac{\pi}{12}\right) - \frac{1}{2}\arcsin(1) = 2 \times \frac{\pi}{12} - \frac{1}{2} \times \frac{\pi}{2} =$

$$= \frac{\pi}{6} - \frac{\pi}{4} = \frac{2\pi - 3\pi}{12} = -\frac{\pi}{12}$$

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4. $h(x) = 1 + 2\cos\left(\frac{x}{3}\right), D_h = \mathbb{R}$

4.1. $x \in \mathbb{R} \Leftrightarrow \frac{x}{3} \in \mathbb{R} \Leftrightarrow -1 \leq \cos\frac{x}{3} \leq 1 \Leftrightarrow$

$$\Leftrightarrow -2 \leq 2\cos\frac{x}{3} \leq 2 \Leftrightarrow$$

$$\Leftrightarrow 1 - 2 \leq 1 + 2\cos\frac{x}{3} \leq 1 + 2 \Leftrightarrow$$

$$\Leftrightarrow -1 \leq h(x) \leq 3$$

$$D'_h = [-1, 3]$$

4.2. $D_h = \mathbb{R}$

Se $x \in D_h$, então $-x \in D_h$, pois $D_h = \mathbb{R}$.

$$\begin{aligned} h(-x) &= 1 + 2\cos\left(\frac{-x}{3}\right) = 1 + 2\cos\left(-\frac{x}{3}\right) = \\ &= 1 + 2\cos\left(\frac{x}{3}\right) = h(x) \end{aligned}$$

$h(-x) = h(x), \forall x \in \mathbb{R}$. Logo, h é uma função par.

4.3. Seja P o período positivo mínimo da função h .

Se $x \in D_h$, então $x + P \in D_h$ porque $D_h = \mathbb{R}$.

$$\forall x \in \mathbb{R}, h(x + P) = h(x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R}, 1 + 2\cos\left(\frac{x + P}{3}\right) = 1 + 2\cos\frac{x}{3} \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R}, 1 + 2\cos\left(\frac{x}{3} + \frac{P}{3}\right) = 1 + 2\cos\frac{x}{3} \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R}, \cos\left(\frac{x}{3} + \frac{P}{3}\right) = \cos\frac{x}{3} \Leftrightarrow$$

$$\Leftrightarrow \frac{P}{3} = 2\pi \text{ porque } 2\pi \text{ é o período positivo mínimo}$$

da função cosseno. Logo, $P = 6\pi$.

Logo, a função h é uma função periódica de período fundamental igual a 6π .

4.4. O gráfico da função h pode ser obtido do gráfico da função $y = \cos x$ por uma dilatação horizontal de coeficiente 3 seguida de uma dilatação vertical de coeficiente 2 e de uma translação de vetor $(0, 1)$.

$$\begin{aligned} 5.1. \quad \arccos\left(-\frac{1}{2}\right) &= x \Leftrightarrow \\ \Leftrightarrow -\frac{1}{2} &= \cos x \wedge x \in [0, \pi] \Leftrightarrow \\ \Leftrightarrow x &= \pi - \frac{\pi}{3} \Leftrightarrow x = \frac{2\pi}{3} \end{aligned}$$

$$\pi - \arccos\left(-\frac{1}{2}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\begin{aligned} 5.2. \quad \arccos\left(\frac{1}{2} - \sin\frac{\pi}{2}\right) + \arccos\left(-\frac{1}{2}\right) &= \\ = \arccos\left(\frac{1}{2} - 1\right) + \frac{2\pi}{3} &= \\ = \arccos\left(-\frac{1}{2}\right) + \frac{2\pi}{3} &= \\ = \frac{2\pi}{3} + \frac{2\pi}{3} &= \frac{4\pi}{3} \end{aligned}$$

$$\begin{aligned} 5.3. \quad \arccos\left(\cos\frac{\pi}{12}\right) + \arccos\left(\sin\left(-\frac{\pi}{4}\right)\right) &= \\ = \frac{\pi}{12} + \arccos\left(-\sin\frac{\pi}{4}\right) &= \\ = \frac{\pi}{12} + \arccos\left(-\frac{\sqrt{2}}{2}\right) &= \\ = \frac{\pi}{12} + \frac{3\pi}{4} &= \frac{5\pi}{6} \end{aligned}$$

Cálculo auxiliar:

$$\begin{aligned} \arccos\left(-\frac{\sqrt{2}}{2}\right) &= x \Leftrightarrow -\frac{\sqrt{2}}{2} = \cos x \wedge x \in [0, \pi] \Leftrightarrow \\ \Leftrightarrow x &= \pi - \frac{\pi}{4} \Leftrightarrow x = \frac{3\pi}{4} \end{aligned}$$

$$5.4. \quad \sin\left(\frac{\pi}{2} - \arccos\frac{\sqrt{3}}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

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$$6. \quad f(x) = a \tan\left(2x - \frac{\pi}{3}\right), \quad a \in \mathbb{R} \setminus \{0\}$$

$$\begin{aligned} 6.1. \quad D_f &= \mathbb{R} \setminus \left\{x: 2x - \frac{\pi}{3} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\} = \\ &= \mathbb{R} \setminus \left\{x: x = \frac{5\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}\right\} \end{aligned}$$

Cálculo auxiliar:

$$\begin{aligned} 2x - \frac{\pi}{3} &= \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow 2x &= \frac{\pi}{2} + \frac{\pi}{3} + k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow 2x &= \frac{5\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{5\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z} \end{aligned}$$

$$6.2. \quad f(x) = 0 \Leftrightarrow a \tan\left(2x - \frac{\pi}{3}\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan\left(2x - \frac{\pi}{3}\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2x - \frac{\pi}{3} = k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

6.3. A função f não é ímpar.

Por exemplo, $f\left(-\frac{\pi}{6}\right) \neq -f\left(\frac{\pi}{6}\right)$ pois $f\left(\frac{\pi}{6}\right) = 0$ e

$$\begin{aligned} f\left(-\frac{\pi}{6}\right) &= a \tan\left(-\frac{2\pi}{6} - \frac{\pi}{3}\right) = \\ &= a \tan\left(-\frac{2\pi}{3}\right) = -a \tan\frac{2\pi}{3} = -a \tan\left(\pi - \frac{\pi}{3}\right) = \\ &= -a \tan\left(-\frac{\pi}{3}\right) = a \tan\frac{\pi}{3} = a\sqrt{3} \neq 0 \end{aligned}$$

6.4. Se $x \in D_f$, então $x + \frac{\pi}{2} \in D_f$.

$$\begin{aligned} \forall x \in D_f, \quad f\left(x + \frac{\pi}{2}\right) &= a \tan\left(2\left(x + \frac{\pi}{2}\right) - \frac{\pi}{3}\right) = \\ &= a \tan\left(2x + \pi - \frac{\pi}{3}\right) = a \tan\left[\pi + \left(2x - \frac{\pi}{3}\right)\right] = \\ &= a \tan\left(2x - \frac{\pi}{3}\right) = f(x) \end{aligned}$$

$$6.5. \quad f\left(\frac{\pi}{3}\right) - 3f\left(\frac{\pi}{12}\right) = 6 \Leftrightarrow$$

$$\Leftrightarrow a \tan\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) - 3a \tan\left(2 \times \frac{\pi}{12} - \frac{\pi}{3}\right) = 6 \Leftrightarrow$$

$$\Leftrightarrow a \tan\frac{\pi}{3} - 3a \tan\left(\frac{\pi}{6} - \frac{\pi}{3}\right) = 6 \Leftrightarrow$$

$$\Leftrightarrow a \times \sqrt{3} - 3a \tan\left(-\frac{\pi}{6}\right) = 6 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3}a + 3a \tan\frac{\pi}{6} = 6 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3}a + 3a \times \frac{\sqrt{3}}{3} = 6 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3}a + \sqrt{3}a = 6 \Leftrightarrow$$

$$\Leftrightarrow 2\sqrt{3}a = 6 \Leftrightarrow$$

$$\Leftrightarrow a = \frac{6}{2\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow a = \frac{6\sqrt{3}}{2 \times \sqrt{3} \times \sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow a = \sqrt{3}$$

$$7.1. \quad \cos(\arctan(-1)) = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Cálculo auxiliar:

$$\arctan(-1) = x \Leftrightarrow -1 = \tan x \wedge x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\Leftrightarrow x = -\frac{\pi}{4}$$

$$7.2. \arctan\left(\sin\frac{\pi}{2}\right) = \arctan(1) = \frac{\pi}{4}$$

$$\begin{aligned} 7.3. \arctan\left(\cos\frac{\pi}{6} - \frac{5\sqrt{3}}{6}\right) &= \arctan\left(\frac{\sqrt{3}}{2} - \frac{5\sqrt{3}}{6}\right) = \\ &= \arctan\left(\frac{3\sqrt{3} - 5\sqrt{3}}{6}\right) = \arctan\left(-\frac{2\sqrt{3}}{6}\right) = \\ &= \arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} \end{aligned}$$

Cálculo auxiliar:

$$\arctan\left(-\frac{\sqrt{3}}{3}\right) = x \Leftrightarrow -\frac{\sqrt{3}}{3} = \tan x \wedge x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\Leftrightarrow x = -\frac{\pi}{6}$$

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$$\begin{aligned} 8.1. 2\sin x - 1 &= 0 \Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow \sin x = \sin\frac{\pi}{6} \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{6} + 2k\pi \vee x = \pi - \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \end{aligned}$$

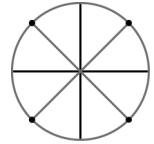
$$\begin{aligned} 8.2. 2\sin\left(x - \frac{\pi}{3}\right) - \sqrt{3} &= 0 \Leftrightarrow \sin\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \\ \Leftrightarrow \sin\left(x - \frac{\pi}{3}\right) &= \sin\frac{\pi}{3} \Leftrightarrow \\ \Leftrightarrow x - \frac{\pi}{3} &= \frac{\pi}{3} + 2k\pi \vee x - \frac{\pi}{3} = \pi - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{2\pi}{3} + 2k\pi \vee x = \pi + 2k\pi, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 8.3. 4\sin\left(2x + \frac{\pi}{4}\right) + \sqrt{8} &= 0 \Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) = -\frac{2\sqrt{2}}{4} \Leftrightarrow \\ \Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) &= -\frac{\sqrt{2}}{2} \Leftrightarrow \sin\left(2x + \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) \Leftrightarrow \\ \Leftrightarrow 2x + \frac{\pi}{4} &= -\frac{\pi}{4} + 2k\pi \vee \\ \vee 2x + \frac{\pi}{4} &= \pi - \left(-\frac{\pi}{4}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow 2x &= -\frac{\pi}{2} + 2k\pi \vee 2x = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= -\frac{\pi}{4} + k\pi \vee x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 8.4. 3\sin x + 2\sqrt{3}\sin^2 x &= 0 \Leftrightarrow \sin x(3 + 2\sqrt{3}\sin x) = 0 \Leftrightarrow \\ \Leftrightarrow \sin x &= 0 \vee 3 + 2\sqrt{3}\sin x = 0 \Leftrightarrow \\ \Leftrightarrow \sin x &= 0 \vee \sin x = -\frac{3}{2\sqrt{3}} \Leftrightarrow \\ \Leftrightarrow \sin x &= 0 \vee \sin x = -\frac{\sqrt{3}}{2} \Leftrightarrow \\ \Leftrightarrow \sin x &= 0 \vee \sin x = \sin\left(-\frac{\pi}{3}\right) \Leftrightarrow \\ \Leftrightarrow x &= k\pi \vee x = -\frac{\pi}{3} + 2k\pi \vee \\ \vee x &= \pi - \left(-\frac{\pi}{3}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= k\pi \vee x = -\frac{\pi}{3} + 2k\pi \vee x = \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 8.5. \sin^2\left(x - \frac{\pi}{4}\right) &= 1 \Leftrightarrow \sin\left(x - \frac{\pi}{4}\right) = 1 \vee \sin\left(x - \frac{\pi}{4}\right) = -1 \Leftrightarrow \\ \Leftrightarrow x - \frac{\pi}{4} &= \frac{\pi}{2} + 2k\pi \vee x - \frac{\pi}{4} = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{3\pi}{4} + 2k\pi \vee x = \frac{7\pi}{4} + 2k\pi, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 8.6. 2\sin^2 x - 1 &= 0 \Leftrightarrow \sin^2 x = \frac{1}{2} \Leftrightarrow \\ \Leftrightarrow \sin x &= \sqrt{\frac{1}{2}} \vee \sin x = -\sqrt{\frac{1}{2}} \Leftrightarrow \\ \Leftrightarrow \sin x &= \frac{\sqrt{2}}{2} \vee \sin x = -\frac{\sqrt{2}}{2} \Leftrightarrow \\ \Leftrightarrow \sin x &= \sin\frac{\pi}{4} \vee \sin x = \sin\left(-\frac{\pi}{4}\right) \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{4} + 2k\pi \vee x = \pi - \frac{\pi}{4} + 2k\pi \vee \\ \vee x &= -\frac{\pi}{4} + 2k\pi \vee x = \pi + \frac{\pi}{4} + 2k\pi \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{4} + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi \vee x = -\frac{\pi}{4} + 2k\pi \vee \\ \vee x &= \frac{5\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z} \end{aligned}$$



$$\begin{aligned} 8.7. 2\sin^2 x - \sin x - 1 &= 0 \Leftrightarrow \\ \Leftrightarrow 2y^2 - y - 1 &= 0 \Leftrightarrow \\ \Leftrightarrow y^2 = \frac{1 \pm \sqrt{1+8}}{4} \Leftrightarrow y &= 1 \vee y = -\frac{1}{2} \Leftrightarrow \\ \Leftrightarrow \sin x &= 1 \vee \sin x = -\frac{1}{2} \Leftrightarrow \\ \Leftrightarrow \sin x &= 1 \vee \sin x = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{2} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi \vee \\ \vee x &= \pi - \left(-\frac{\pi}{6}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{2} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 8.8. 4\sin^2(3x) &= 8\sin(3x) - 3 \Leftrightarrow \\ \Leftrightarrow 4\sin^2(3x) - 8\sin(3x) + 3 &= 0 \Leftrightarrow \\ \Leftrightarrow 4y^2 - 8y + 3 &= 0 \Leftrightarrow \\ \Leftrightarrow y &= \frac{8 \pm \sqrt{64 - 4 \times 4 \times 3}}{8} \Leftrightarrow \\ \Leftrightarrow y &= \frac{1}{2} \vee y = \frac{3}{2} \Leftrightarrow \\ \Leftrightarrow \sin(3x) &= \frac{1}{2} \vee \sin(3x) = \frac{3}{2} \Leftrightarrow \\ \Leftrightarrow \sin(3x) &= \sin\frac{\pi}{6} \vee \text{condição impossível} \Leftrightarrow \\ \Leftrightarrow 3x &= \frac{\pi}{6} + 2k\pi \vee 3x = \pi - \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\ \Leftrightarrow x &= \frac{\pi}{18} + \frac{2k\pi}{3} \vee x = \frac{5\pi}{18} + \frac{2k\pi}{3}, k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned}
 8.9. \quad \sin^3 x - 2\sin^2 x &= 0 \Leftrightarrow \\
 &\Leftrightarrow \sin^2 x (\sin x - 2) = 0 \Leftrightarrow \\
 &\Leftrightarrow \sin^2 x = 0 \vee \sin x - 2 = 0 \Leftrightarrow \\
 &\Leftrightarrow \sin x = 0 \vee \sin x = 2 \Leftrightarrow \\
 &\Leftrightarrow x = k\pi, k \in \mathbb{Z} \vee \text{condição impossível} \Leftrightarrow \\
 &\Leftrightarrow x = k\pi, k \in \mathbb{Z}
 \end{aligned}$$

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$$\begin{aligned}
 9.1. \quad \sqrt{2}\sin x - 1 &= 0 \wedge x \in [0, 2\pi[\\
 \sqrt{2}\sin x - 1 &= 0 \Leftrightarrow \\
 \Leftrightarrow \sin x &= \frac{1}{\sqrt{2}} \Leftrightarrow \sin x = \frac{\sqrt{2}}{2} \Leftrightarrow \\
 \Leftrightarrow \sin x &= \sin \frac{\pi}{4} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{4} + 2k\pi \vee x = \pi - \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{4} + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z}
 \end{aligned}$$

Atribuindo os valores a k , obtemos as soluções do intervalo $[0, 2\pi[$:

$$\begin{aligned}
 k=0 \Rightarrow x &= \frac{\pi}{4} \vee x = \frac{3\pi}{4} \\
 S &= \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}
 \end{aligned}$$

$$\begin{aligned}
 9.2. \quad 2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) - 3 &= 0 \wedge x \in [-\pi, \pi[\\
 2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) - 3 &= 0 \Leftrightarrow \\
 \Leftrightarrow 2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) &= 3 \Leftrightarrow \\
 \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) &= \frac{3}{2\sqrt{3}} \Leftrightarrow \\
 \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) &= \frac{\sqrt{3}}{2} \Leftrightarrow \\
 \Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) &= \sin \frac{\pi}{3} \Leftrightarrow \\
 \Leftrightarrow x + \frac{\pi}{4} &= \frac{\pi}{3} + 2k\pi \vee x + \frac{\pi}{4} = \pi - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi \vee x = \frac{2\pi}{3} - \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{12} + 2k\pi \vee x = \frac{5\pi}{12} + 2k\pi, k \in \mathbb{Z}
 \end{aligned}$$

Soluções no intervalo $[-\pi, \pi[$:

$$\begin{aligned}
 x &= \frac{\pi}{12} + 2k\pi \\
 k=-1 \Rightarrow x &= \frac{\pi}{12} - 2\pi \Leftrightarrow x = -\frac{23\pi}{12} \quad \times \\
 k=0 \Rightarrow x &= \frac{\pi}{12} \quad \checkmark \\
 k=1 \Rightarrow x &= \frac{\pi}{12} + 2\pi \quad \times
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{5\pi}{12} + 2k\pi \\
 k=-1 \Rightarrow x &= \frac{5\pi}{12} - 2\pi \Leftrightarrow x = -\frac{19\pi}{12} \quad \times \\
 k=0 \Rightarrow x &= \frac{5\pi}{12} \quad \checkmark \\
 k=1 \Rightarrow x &= \frac{5\pi}{12} + 2\pi \quad \times
 \end{aligned}$$

$$S = \left\{ \frac{\pi}{12}, \frac{5\pi}{12} \right\}$$

$$\begin{aligned}
 9.3. \quad \sin\left(x + \frac{\pi}{3}\right) - \cos x &= 0 \wedge x \in [-\pi, \pi[\\
 \sin\left(x + \frac{\pi}{3}\right) - \cos x &= 0 \Leftrightarrow \\
 \Leftrightarrow \sin\left(x + \frac{\pi}{3}\right) &= \cos x \Leftrightarrow \\
 \Leftrightarrow \sin\left(x + \frac{\pi}{3}\right) &= \sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow \\
 \Leftrightarrow x + \frac{\pi}{3} &= \frac{\pi}{2} - x + 2k\pi \vee \\
 \vee x + \frac{\pi}{3} &= \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow 2x &= \frac{\pi}{6} + 2k\pi \vee x - x = \frac{\pi}{2} - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{12} + k\pi, k \in \mathbb{Z} \vee \text{condição impossível} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{12} + k\pi, k \in \mathbb{Z}
 \end{aligned}$$

Soluções no intervalo $[-\pi, \pi[$:

$$\begin{aligned}
 k=-1 \Rightarrow x &= \frac{\pi}{12} - \pi \Leftrightarrow x = -\frac{11\pi}{12} \quad \checkmark \\
 k=-2 \Rightarrow x &= \frac{\pi}{12} - 2\pi \Leftrightarrow x = -\frac{23\pi}{12} \quad \times \\
 k=0 \Rightarrow x &= \frac{\pi}{12} \quad \checkmark \\
 k=1 \Rightarrow x &= \frac{\pi}{12} + \pi \quad \times
 \end{aligned}$$

$$S = \left\{ -\frac{11\pi}{12}, \frac{\pi}{12} \right\}$$

$$\begin{aligned}
 9.4. \quad \sin^2(\pi x) - 1 &= 0 \wedge x \in [0, 2] \\
 \sin^2(\pi x) - 1 &= 0 \Leftrightarrow \sin^2(\pi x) = 1 \Leftrightarrow \\
 \Leftrightarrow \sin(\pi x) &= 1 \vee \sin(\pi x) = -1 \Leftrightarrow \\
 \Leftrightarrow \pi x &= \frac{\pi}{2} + 2k\pi \vee \pi x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{1}{2} + 2k \vee x = \frac{3}{2} + 2k, k \in \mathbb{Z}
 \end{aligned}$$

As soluções que pertencem ao intervalo $[0, 2]$ são as que se obtém para $k=0$, ou seja, $S = \left\{ \frac{1}{2}, \frac{3}{2} \right\}$.

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$$\begin{aligned}
 10.1. \quad 2\cos x - 1 &= 0 \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow \cos x = \cos \frac{\pi}{3} \Leftrightarrow \\
 \Leftrightarrow x &= \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned} \mathbf{10.2.} \quad 2 \cos x + 1 &= 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow \cos x = -\cos \frac{\pi}{3} \Leftrightarrow \\ &\Leftrightarrow \cos x = \cos \left(\pi - \frac{\pi}{3} \right) \Leftrightarrow \cos x = \cos \frac{2\pi}{3} \Leftrightarrow \\ &\Leftrightarrow x = \frac{2\pi}{3} + 2k\pi \vee x = -\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \mathbf{10.3.} \quad & 2 \cos\left(x - \frac{\pi}{3}\right) = \sqrt{2} \Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \Leftrightarrow \\ & \Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos \frac{\pi}{4} \Leftrightarrow \\ & \Leftrightarrow x - \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi \vee x - \frac{\pi}{3} = -\frac{\pi}{4} + 2k\pi, \quad k \in \mathbb{Z} \Leftrightarrow \\ & \Leftrightarrow x = \frac{\pi}{4} + \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{4} + \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \Leftrightarrow \\ & \Leftrightarrow x = \frac{7\pi}{12} + 2k\pi \vee x = \frac{\pi}{12} + 2k\pi, \quad k \in \mathbb{Z} \end{aligned}$$

10.4. $\sqrt{8} \cos\left(2x + \frac{\pi}{4}\right) + \sqrt{6} = 0 \Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{6}}{\sqrt{8}} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\sqrt{\frac{3}{4}} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = \cos\frac{5\pi}{6} \Leftrightarrow$

$\Leftrightarrow 2x + \frac{\pi}{4} = \frac{5\pi}{6} + 2k\pi \vee 2x + \frac{\pi}{4} = -\frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{5\pi}{6} - \frac{\pi}{4} + 2k\pi \vee 2x = -\frac{5\pi}{6} - \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{7\pi}{12} + 2k\pi \vee 2x = -\frac{13\pi}{12} + 2k\pi, k \in \mathbb{Z}$

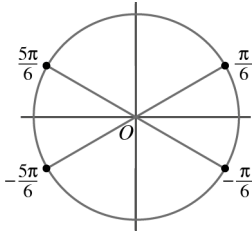
$\Leftrightarrow x = \frac{7\pi}{24} + k\pi \vee x = -\frac{13\pi}{24} + k\pi, k \in \mathbb{Z}$

10.5. $\cos^2 x = \frac{3}{4} \Leftrightarrow$

$$\Leftrightarrow \cos x = \frac{\sqrt{3}}{2} \vee \cos x = -\frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos \frac{\pi}{6} \vee \cos x = \cos \left(\pi - \frac{\pi}{6} \right) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee$$

$$\vee x = -\frac{5\pi}{6} + 2k\pi \Leftrightarrow$$


$$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = -\frac{\pi}{6} + k\pi, \quad k \in \mathbb{Z}$$

10.6. $2 \cos^2 x + \cos x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow 2y^2 + y - 1 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{-1 \pm \sqrt{1+8}}{4} \Leftrightarrow$

$\Leftrightarrow y = -1 \vee y = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = -1 \vee \cos x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = -1 \vee \cos x = \cos \frac{\pi}{3} \Leftrightarrow$

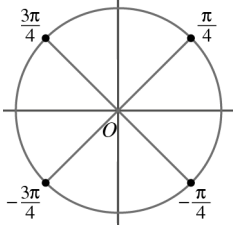
$\Leftrightarrow x = \pi + 2k\pi \vee x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$$\begin{aligned} \mathbf{10.7.} \quad & \cos^2\left(x - \frac{\pi}{8}\right) - 1 = 0 \Leftrightarrow \\ & \Leftrightarrow \cos^2\left(x - \frac{\pi}{8}\right) = 1 \Leftrightarrow \\ & \Leftrightarrow \cos\left(x - \frac{\pi}{8}\right) = -1 \vee \cos\left(x - \frac{\pi}{8}\right) = 1 \Leftrightarrow \\ & \Leftrightarrow x - \frac{\pi}{8} = \pi + 2k\pi \vee x - \frac{\pi}{8} = 2k\pi, \quad k \in \mathbb{Z} \Leftrightarrow \\ & \Leftrightarrow x = \frac{9\pi}{8} + 2k\pi \vee x = \frac{\pi}{8} + 2k\pi, \quad k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \mathbf{10.8.} \quad & \left(\sqrt{2} \cos x + 1\right) \left(\frac{\sin x}{2} - 1\right) = 0 \Leftrightarrow \\ & \Leftrightarrow \sqrt{2} \cos x + 1 = 0 \vee \frac{\sin x}{2} - 1 = 0 \Leftrightarrow \\ & \Leftrightarrow \cos x = -\frac{1}{\sqrt{2}} \vee \sin x - 2 = 0 \Leftrightarrow \\ & \Leftrightarrow \cos x = -\frac{\sqrt{2}}{2} \vee \sin x = 2 \Leftrightarrow \\ & \Leftrightarrow \cos x = \cos\left(\pi - \frac{\pi}{4}\right) \vee \text{condição impossível} \Leftrightarrow \\ & \Leftrightarrow x = \frac{3\pi}{4} + 2k\pi \vee x = -\frac{3\pi}{4} + 2k\pi, \quad k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned}
 10.9. \quad & 4\sin^2 x + 8\cos x - 7 = 0 \Leftrightarrow \\
 & \Leftrightarrow 4(1 - \cos^2 x) + 8\cos x - 7 = 0 \Leftrightarrow \\
 & \Leftrightarrow 4 - 4\cos^2 x + 8\cos x - 7 = 0 \Leftrightarrow \\
 & \Leftrightarrow 4\cos^2 x - 8\cos x + 3 = 0 \Leftrightarrow \\
 & \Leftrightarrow 4y^2 - 8y + 3 = 0 \Leftrightarrow \\
 & \Leftrightarrow y = \frac{8 \pm \sqrt{64 - 4 \times 4 \times 3}}{8} \Leftrightarrow \\
 & \Leftrightarrow y = \frac{1}{2} \vee y = \frac{3}{2} \Leftrightarrow \\
 & \Leftrightarrow \cos x = \frac{1}{2} \vee \cos x = \frac{3}{2} \\
 & \Leftrightarrow \cos x = \cos \frac{\pi}{3} \vee \text{condição impossível} \Leftrightarrow \\
 & \Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}
 \end{aligned}$$

10.10. $\cos^2(2x) = \sin^2(2x) \Leftrightarrow$
 $\Leftrightarrow \cos^2(2x) = 1 - \cos^2(2x) \Leftrightarrow$
 $\Leftrightarrow 2\cos^2(2x) = 1 \Leftrightarrow$
 $\Leftrightarrow \cos^2(2x) = \frac{1}{2} \Leftrightarrow$
 $\Leftrightarrow \cos(2x) = -\sqrt{\frac{1}{2}} \vee \cos(2x) = \sqrt{\frac{1}{2}} \Leftrightarrow$
 $\Leftrightarrow \cos(2x) = -\frac{\sqrt{2}}{2} \vee \cos(2x) = \frac{\sqrt{2}}{2} \Leftrightarrow$
 $\Leftrightarrow \cos(2x) = \cos\left(\pi - \frac{\pi}{4}\right) \vee \cos(2x) = \cos\frac{\pi}{4} \Leftrightarrow$
 $\Leftrightarrow 2x = -\frac{3\pi}{4} + 2k\pi \vee 2x = \frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$
 $\Leftrightarrow 2x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z} \Leftrightarrow$



$\Leftrightarrow x = \frac{\pi}{8} + \frac{k\pi}{4}, k \in \mathbb{Z}$

11.1. $2\cos x + \sqrt{3} = 0 \wedge x \in]-\pi, \pi]$

$$2\cos x + \sqrt{3} = 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = -\frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos\frac{5\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{5\pi}{6} + 2k\pi \vee x = -\frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$$

As únicas soluções que pertencem ao intervalo $]-\pi, \pi]$ são

as que se obtêm para $k = 0$, ou seja, $S = \left\{-\frac{5\pi}{6}, \frac{5\pi}{6}\right\}$.

11.2. $\cos x - \sqrt{0,5} = 0 \wedge x \in [0, 2\pi]$

$$\cos x - \sqrt{0,5} = 0 \Leftrightarrow \cos x = \sqrt{\frac{1}{2}} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{4} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{4} + 2k\pi \vee x = \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$$

Atribuindo valores a k , obtemos:

$$x = -\frac{\pi}{4} + 2k\pi$$

$$k = 0 \Rightarrow x = -\frac{\pi}{4} \quad \times$$

$$k = 1 \Rightarrow x = \frac{7\pi}{4} \quad \checkmark$$

$$k = 2 \Rightarrow x = -\frac{\pi}{4} + 4\pi \quad \times$$

$$x = \frac{\pi}{4} + 2k\pi$$

$$k = 0 \Rightarrow x = \frac{\pi}{4} \quad \checkmark$$

$$k = 1 \Rightarrow x = \frac{\pi}{4} + 2\pi \quad \times$$

$$S = \left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$$

11.3. $8\cos\left(\frac{\pi x}{2}\right) + \sqrt{32} = 0 \wedge x \in [-2, 2]$

$$8\cos\left(\frac{\pi x}{2}\right) + \sqrt{32} = 0 \Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\frac{\sqrt{32}}{8} \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\sqrt{\frac{32}{64}} \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\sqrt{\frac{1}{2}} \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\frac{\sqrt{2}}{2} \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = \cos\left(\pi - \frac{\pi}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi x}{2} = \frac{3\pi}{4} + 2k\pi \vee \frac{\pi x}{2} = -\frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2\pi x = 3\pi + 8k\pi \vee 2\pi x = -3\pi + 8k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = 3 + 8k \vee 2x = -3 + 8k, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3}{2} + 4k \vee x = -\frac{3}{2} + 4k, k \in \mathbb{Z}$$

Atribuindo valores a k obtemos as soluções que pertencem ao intervalo $[-2, 2]$.

$$k = 0 \Rightarrow x = \frac{3}{2} \vee x = -\frac{3}{2} \quad \checkmark$$

$$k = -1 \Rightarrow x = \frac{3}{2} - 4 \vee x = -\frac{3}{2} - 4 \quad \times$$

$$k = 1 \Rightarrow x = \frac{3}{2} + 4 \vee x = -\frac{3}{2} + 4 \quad \times$$

$$S = \left\{-\frac{3}{2}, \frac{3}{2}\right\}$$

11.4. $\cos x + \sin x = 0 \wedge x \in]-\pi, \pi[$

$$\cos x + \sin x = 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = -\sin x \Leftrightarrow$$

$$\Leftrightarrow \cos x = \sin(-x) \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} - (-x)\right) \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} + x\right) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} + x + 2k\pi \vee x = -\frac{\pi}{2} - x + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \text{Equação impossível} \vee 2x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$k = -1 \Rightarrow x = -\frac{\pi}{4} - \pi \quad \times$$

$$k = 0 \Rightarrow x = -\frac{\pi}{4} \quad \checkmark$$

$$k=1 \Rightarrow x = -\frac{\pi}{4} + \pi = \frac{3\pi}{4} \quad \checkmark$$

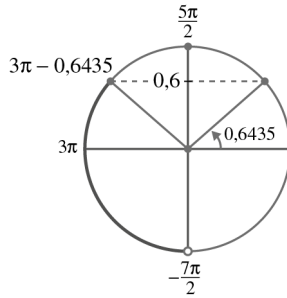
$$k=2 \Rightarrow x = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4} \quad \times$$

$$S = \left\{ -\frac{\pi}{4}, \frac{3\pi}{4} \right\}$$

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$$12.1. \sin x = 0,6 \wedge x \in \left[\frac{5\pi}{2}, \frac{7\pi}{2} \right]$$

Recorrendo à calculadora, obtemos, $\arcsin(0,6) \approx 0,6435$.



$$\sin x = 0,6 \wedge x \in \left[\frac{5\pi}{2}, \frac{7\pi}{2} \right] \Rightarrow$$

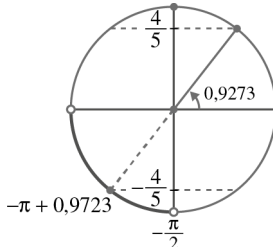
$$\Rightarrow x \approx (3\pi - 0,6435) \text{ rad}$$

$$\Rightarrow x \approx 8,78 \text{ rad}$$

$$12.2. 5\sin(3x) + 4 = 0 \wedge x \in \left[-\frac{\pi}{3}, -\frac{\pi}{6} \right] \Leftrightarrow$$

$$\Leftrightarrow \sin(3x) = -\frac{4}{5} \wedge 3x \in \left[-\pi, -\frac{\pi}{2} \right]$$

Recorrendo à calculadora, obtemos $\arcsin\left(\frac{4}{5}\right) \approx 0,9273$.



$$\sin(3x) = -\frac{4}{5} \wedge 3x \in \left[-\pi, -\frac{\pi}{2} \right] \Rightarrow$$

$$\Rightarrow 3x \approx (-\pi + 0,9273) \text{ rad} \Rightarrow$$

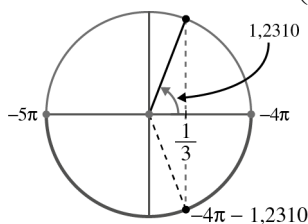
$$\Rightarrow 3x \approx -2,2143 \text{ rad} \Rightarrow$$

$$\Rightarrow x \approx -0,74 \text{ rad}$$

$$12.3. 3\cos(2x) = 1 \wedge x \in \left[-\frac{5\pi}{2}, -2\pi \right] \Leftrightarrow$$

$$\Leftrightarrow \cos(2x) = \frac{1}{3} \wedge 2x \in [-5\pi, -4\pi]$$

Recorrendo à calculadora obtemos $\arccos\left(\frac{1}{3}\right) \approx 1,2310$.



$$\cos(2x) = \frac{1}{3} \wedge 2x \in [-5\pi, -4\pi] \Rightarrow$$

$$\Rightarrow 2x \approx (-4\pi - 1,2310) \text{ rad} \Rightarrow$$

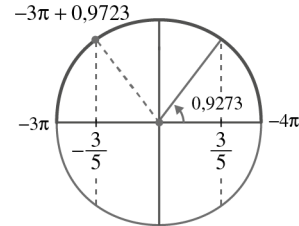
$$\Rightarrow 2x \approx -13,7974 \text{ rad} \Rightarrow x \approx -6,90 \text{ rad}$$

$$12.4. 2\cos\frac{x}{2} + \frac{6}{5} = 0 \wedge x \in [-8\pi, -6\pi] \Leftrightarrow$$

$$\Leftrightarrow 2\cos\frac{x}{2} = -\frac{6}{5} \wedge \frac{x}{2} \in [-4\pi, -3\pi] \Leftrightarrow$$

$$\Leftrightarrow \cos\frac{x}{2} = -\frac{3}{5} \wedge \frac{x}{2} \in [-4\pi, -3\pi]$$

Na calculadora obtemos $\arccos\left(\frac{3}{5}\right) \approx 0,9273$.



$$\cos\frac{x}{2} = -\frac{3}{5} \wedge \frac{x}{2} \in [-4\pi, -3\pi] \Rightarrow$$

$$\Rightarrow \frac{x}{2} \approx (-3\pi - 0,9273) \text{ rad} \Rightarrow$$

$$\Rightarrow \frac{x}{2} \approx -10,3521 \text{ rad} \Rightarrow x \approx -20,70 \text{ rad}$$

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$$13.1. \tan(\pi x) + 1 = 0 \wedge x \in [0, 1]$$

$$\tan(\pi x) + 1 = 0 \Leftrightarrow \tan(\pi x) = -1 \Leftrightarrow$$

$$\Leftrightarrow \tan(\pi x) = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow \pi x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{1}{4} + k, k \in \mathbb{Z}$$

Para $k=1$ obtemos $x = \frac{3}{4}$ que é a única solução do intervalo

$$[0, 1], \text{ ou seja, } S = \left\{ \frac{3}{4} \right\}.$$

$$13.2. 6\tan\left(2x - \frac{\pi}{6}\right) - \sqrt{12} = 0 \wedge x \in \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]$$

$$6\tan\left(2x - \frac{\pi}{6}\right) - \sqrt{12} = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{12}}{6} \Leftrightarrow$$

$$\Leftrightarrow \tan\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \Leftrightarrow \frac{\sqrt{12}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

$$\Leftrightarrow \tan\left(2x - \frac{\pi}{6}\right) = \tan\frac{\pi}{6} \Leftrightarrow 2x - \frac{\pi}{6} = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = 2 \times \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$k=-1 \Rightarrow x = \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3} \quad \times$$

$$k=0 \Rightarrow x = \frac{\pi}{6} \quad \checkmark$$

$$k=1 \Rightarrow x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3} \quad \times$$

$$S = \left\{ \frac{\pi}{6} \right\}$$

$$14.1. 3 \tan\left(2x - \frac{\pi}{6}\right) = 0 \Leftrightarrow \tan\left(2x - \frac{\pi}{6}\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2x - \frac{\pi}{6} = k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$14.2. 3 \tan^2\left(\frac{\pi x}{2}\right) = 1 \Leftrightarrow \tan^2\left(\frac{\pi x}{2}\right) = \frac{1}{3} \Leftrightarrow$$

$$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right) = \pm \sqrt{\frac{1}{3}} \Leftrightarrow$$

$$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right) = \frac{\sqrt{3}}{3} \vee \tan\left(\frac{\pi x}{2}\right) = -\frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right) = \tan\frac{\pi}{6} \vee \tan\left(\frac{\pi x}{2}\right) = \tan\left(-\frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi x}{2} = \frac{\pi}{6} + k\pi \vee \frac{\pi x}{2} = -\frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 3\pi x = \pi + 6k\pi \vee 3\pi x = -\pi + 6k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{1}{3} + 2k \vee x = -\frac{1}{3} + 2k, k \in \mathbb{Z}$$

$$14.3. \tan(2x) = \tan\left(\frac{\pi}{6} - x\right) \Leftrightarrow 2x = \frac{\pi}{6} - x + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 3x = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{18} + \frac{k\pi}{3}, k \in \mathbb{Z}$$

$$14.4. \tan x + \sin x = 0 \Leftrightarrow \frac{\sin x}{\cos x} + \sin x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x \left(\frac{1}{\cos x} + 1\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \frac{1}{\cos x} + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \frac{1}{\cos x} = -1 \Leftrightarrow$$

$$\Leftrightarrow (\sin x = 0 \vee \cos x = -1) \wedge \cos x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow (x = k\pi \vee x = \pi + 2k\pi) \wedge x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = k\pi, k \in \mathbb{Z}$$

$$14.5. \sin x = \tan^2 x \sin x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x - \tan^2 x \sin x = 0 \Leftrightarrow$$

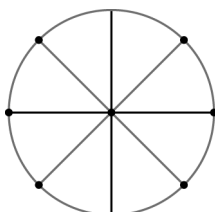
$$\Leftrightarrow \sin x (1 - \tan^2 x) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \tan^2 x = 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \tan x = 1 \vee \tan x = -1 \Leftrightarrow$$

$$\Leftrightarrow \left(x = k\pi \vee x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi\right) \wedge$$

$$\wedge x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$



$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$14.6. 3 \tan^2 x + 2\sqrt{3} \tan x = 3$$

$$\Leftrightarrow 3y^2 + 2\sqrt{3}y - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow y = \frac{-2\sqrt{3} \pm \sqrt{12 + 36}}{6} \Leftrightarrow y = \frac{-2\sqrt{3} \pm \sqrt{48}}{6} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6} \Leftrightarrow y = \frac{-6\sqrt{3}}{6} \vee y = \frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow y = -\sqrt{3} \vee y = \frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow \tan x = -\sqrt{3} \vee \tan x = \frac{\sqrt{3}}{3} \Leftrightarrow$$

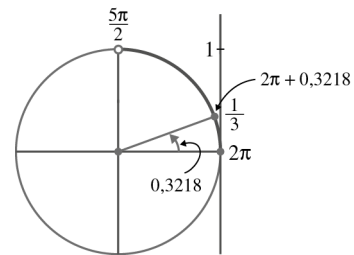
$$\Leftrightarrow \tan x = \tan\left(-\frac{\pi}{3}\right) \vee \tan x = \tan\frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{3} + k\pi \vee x = \frac{\pi}{6} + k\pi, k \in \mathbb{Z}$$

$$15.1. 3 \tan(2x) = 1 \wedge x \in \left[\pi, \frac{5\pi}{4}\right] \Leftrightarrow$$

$$\Leftrightarrow \tan(2x) = \frac{1}{3} \wedge 2x \in \left[2\pi, \frac{5\pi}{2}\right]$$

Na calculadora obtém-se $\arctan\left(\frac{1}{3}\right) \approx 0,3218$.



$$\tan(2x) = \frac{1}{3} \wedge 2x \in \left[2\pi, \frac{5\pi}{2}\right] \Rightarrow$$

$$\Rightarrow 2x \approx 2\pi + 0,3218 \text{ rad} \Rightarrow$$

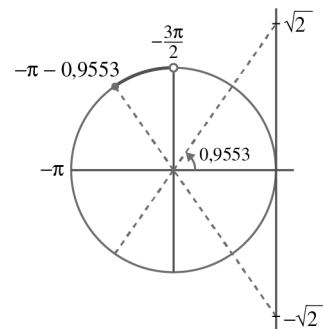
$$\Rightarrow 2x \approx 6,6050 \text{ rad} \Rightarrow$$

$$\Rightarrow x \approx 3,30 \text{ rad}$$

$$15.2. \tan \frac{x}{2} + \sqrt{2} = 0 \wedge x \in]-3\pi, -2\pi] \Leftrightarrow$$

$$\Leftrightarrow \tan \frac{x}{2} = -\sqrt{2} \wedge \frac{x}{2} \in \left]-\frac{3\pi}{2}, -\pi\right]$$

Na calculadora obtemos $\arctan(\sqrt{2}) \approx 0,9553$.



$$\tan \frac{x}{2} = -\sqrt{2} \wedge \frac{x}{2} \in \left]-\frac{3\pi}{2}, -\pi\right] \Rightarrow$$

$$\Rightarrow \frac{x}{2} \approx (-\pi - 0,9553) \text{ rad} \Rightarrow \frac{x}{2} \approx -4,0969 \text{ rad} \Rightarrow$$

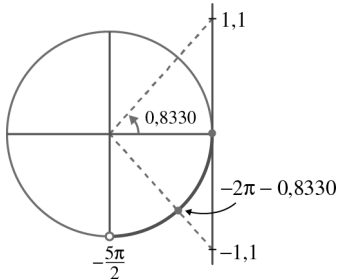
$$\Rightarrow x \approx -8,19 \text{ rad}$$

15.3. $\tan^2 x = 1,21 \wedge x \in \left] -\frac{5\pi}{2}, -2\pi \right] \Leftrightarrow$

$\Leftrightarrow \tan x = -\sqrt{1,21} \wedge x \in \left] -\frac{5\pi}{2}, -2\pi \right] \Leftrightarrow$

$\Leftrightarrow \tan x = -1,1 \wedge x \in \left] -\frac{5\pi}{2}, -2\pi \right]$

Na calculadora obtém-se $\arctan(1,1) \approx 0,8330$.



$\tan x = -1,1 \wedge x \in \left] -\frac{5\pi}{2}, -2\pi \right] \Rightarrow$

$\Rightarrow x \approx (-2\pi - 0,8330) \text{ rad} \Rightarrow x \approx -7,1 \text{ rad}$

$x \in 4.^\circ Q$
 $\tan x < 0$

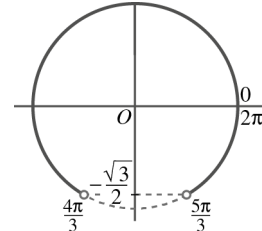
16.3. $4\sin x + \sqrt{12} > 0 \wedge x \in [0, 2\pi[\Leftrightarrow$

$\Leftrightarrow \sin x > -\frac{\sqrt{12}}{4} \wedge x \in [0, 2\pi[\Leftrightarrow$

$\Leftrightarrow \sin x > -\frac{\sqrt{3}}{2} \wedge x \in [0, 2\pi[$

$\sin x = -\frac{\sqrt{3}}{2} \wedge x \in [0, 2\pi[\Leftrightarrow x = \pi + \frac{\pi}{3} \vee x = 2\pi - \frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = \frac{4\pi}{3} \vee x = \frac{5\pi}{3}$



$\sin x > -\frac{\sqrt{3}}{2} \wedge x \in [0, 2\pi[\Leftrightarrow$

$\Leftrightarrow x \in \left[0, \frac{4\pi}{3} \right[\cup \left] \frac{5\pi}{3}, 2\pi \right[$

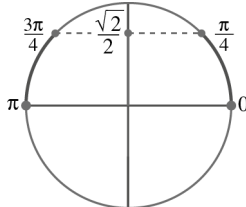
16.1. $2\sin x - \sqrt{2} \leq 0 \wedge x \in [0, \pi] \Leftrightarrow$

$\Leftrightarrow \sin x \leq \frac{\sqrt{2}}{2} \wedge x \in [0, \pi] \Leftrightarrow$

$\sin x = \frac{\sqrt{2}}{2} \wedge x \in [0, \pi] \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} \vee x = \pi - \frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} \vee x = \frac{3\pi}{4}$



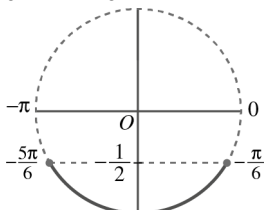
$\sin x \leq \frac{\sqrt{2}}{2} \wedge x \in [0, \pi] \Leftrightarrow x \in \left[0, \frac{\pi}{4} \right] \cup \left[\frac{3\pi}{4}, \pi \right]$

$2\sin x + 1 \leq 0 \wedge x \in [-\pi, \pi] \Leftrightarrow$

16.2. $\Leftrightarrow \sin x \leq -\frac{1}{2} \wedge x \in [-\pi, \pi]$

$\sin x = -\frac{1}{2} \wedge x \in [-\pi, \pi] \Leftrightarrow x = -\pi + \frac{\pi}{6} \vee x = -\frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = -\frac{5\pi}{6} \vee x = -\frac{\pi}{6}$

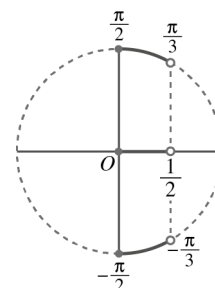


$\sin x \leq -\frac{1}{2} \wedge x \in [-\pi, \pi] \Leftrightarrow x \in \left[-\frac{5\pi}{6}, -\frac{\pi}{6} \right]$

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17.1. $2\cos x < 1 \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \Leftrightarrow \cos x < \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

$\cos x = \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \Leftrightarrow x = -\frac{\pi}{3} \vee x = \frac{\pi}{3}$



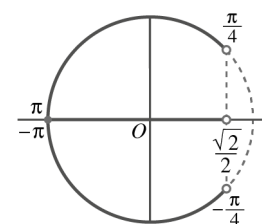
$\cos x < \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \Leftrightarrow x \in \left[-\frac{\pi}{2}, -\frac{\pi}{3} \right[\cup \left] \frac{\pi}{3}, \frac{\pi}{2} \right]$

17.2. $\sqrt{2} - 2\cos x > 0 \wedge x \in [-\pi, \pi] \Leftrightarrow$

$\Leftrightarrow 2\cos x < \sqrt{2} \wedge x \in [-\pi, \pi] \Leftrightarrow$

$\Leftrightarrow \cos x < \frac{\sqrt{2}}{2} \wedge x \in [-\pi, \pi]$

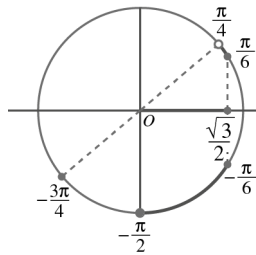
$\cos x = \frac{\sqrt{2}}{2} \wedge x \in [-\pi, \pi] \Leftrightarrow x = -\frac{\pi}{4} \vee x = \frac{\pi}{4}$



$\cos x < \frac{\sqrt{2}}{2} \wedge x \in [-\pi, \pi] \Leftrightarrow x \in \left[-\pi, -\frac{\pi}{4} \right[\cup \left] \frac{\pi}{4}, \pi \right]$

$$17.3. 0 \leq \cos x \leq \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$$

$$\cos x = \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \Leftrightarrow x = -\frac{\pi}{6} \vee x = \frac{\pi}{6}$$

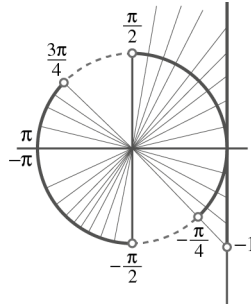


$$0 \leq \cos x \leq \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \Leftrightarrow$$

$$\Leftrightarrow x \in \left[-\frac{\pi}{2}, -\frac{\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$$

$$17.4. \tan x > -1 \wedge x \in]-\pi, \pi]$$

$$\tan x = -1 \wedge x \in]-\pi, \pi] \Leftrightarrow x = -\frac{\pi}{4} \vee x = \frac{3\pi}{4}$$



$$\tan x > -1 \wedge x \in]-\pi, \pi] \Leftrightarrow$$

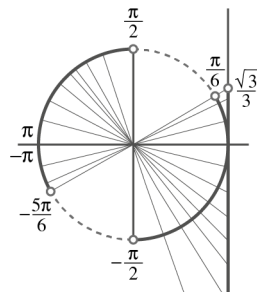
$$\Leftrightarrow x \in \left]-\pi, -\frac{\pi}{2}\right[\cup \left]-\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left[\frac{3\pi}{4}, \pi\right]$$

$$17.5. 3 \tan(2x) < \sqrt{3} \wedge x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow \tan(2x) < \frac{\sqrt{3}}{3} \wedge 2x \in]-\pi, \pi]$$

$$\tan(2x) = \frac{\sqrt{3}}{3} \wedge 2x \in]-\pi, \pi] \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{6} \vee 2x = -\pi + \frac{\pi}{6} \Leftrightarrow 2x = \frac{\pi}{6} \vee 2x = -\frac{5\pi}{6}$$



$$\tan(2x) < \frac{\sqrt{3}}{3} \wedge 2x \in]-\pi, \pi] \Leftrightarrow$$

$$\Leftrightarrow 2x \in \left]-\pi, -\frac{5\pi}{6}\right[\cup \left]-\frac{\pi}{2}, \frac{\pi}{6}\right[\cup \left[\frac{\pi}{2}, \pi\right] \Leftrightarrow$$

$$\Leftrightarrow x \in \left]-\frac{\pi}{2}, -\frac{5\pi}{12}\right[\cup \left]-\frac{\pi}{4}, \frac{\pi}{12}\right[\cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

$$17.6. \tan^2 x < 3 \wedge x \in [0, 2\pi[$$

$$y^2 < 3 \Leftrightarrow y^2 - 3 < 0 \Leftrightarrow -\sqrt{3} < y < \sqrt{3} \quad \text{tan } x = y$$

$$\text{Logo, } \tan^2 x < 3 \wedge x \in [0, 2\pi[\Leftrightarrow$$

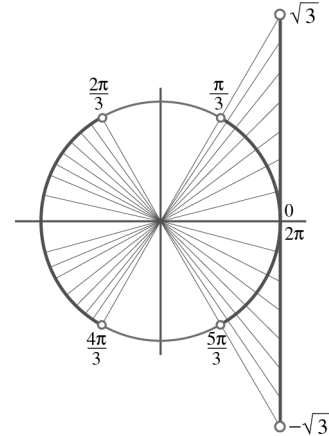
$$\Leftrightarrow -\sqrt{3} < \tan x < \sqrt{3} \wedge x \in [0, 2\pi[$$

No intervalo $[0, 2\pi[$:

$$\tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \pi + \frac{\pi}{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{4\pi}{3}$$

$$\tan x = -\sqrt{3} \Leftrightarrow x = \pi - \frac{\pi}{3} \vee x = 2\pi - \frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{2\pi}{3} \vee x = \frac{5\pi}{3}$$



$$-\sqrt{3} < \tan x < \sqrt{3} \wedge x \in [0, 2\pi[\Leftrightarrow$$

$$\Leftrightarrow x \in \left[0, \frac{\pi}{3}\right[\cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right[\cup \left[\frac{5\pi}{3}, 2\pi\right[$$

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$$18. f(x) = 1 - \sin \frac{x}{2}; g(x) = 3 - 2 \cos^2 \frac{x}{2}; D_f = D_g = [0, 4\pi]$$

$$18.1. f(x) = g(x) \Leftrightarrow 1 - \sin \left(\frac{x}{2}\right) = 3 - 2 \cos^2 \left(\frac{x}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow 2 \cos^2 \frac{x}{2} - \sin \frac{x}{2} - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2 \left(1 - \sin^2 \frac{x}{2}\right) - \sin \frac{x}{2} - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2 - 2 \sin^2 \frac{x}{2} - \sin \frac{x}{2} - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2 \sin^2 \frac{x}{2} + \sin \frac{x}{2} = 0 \Leftrightarrow \sin \frac{x}{2} \left(2 \sin \frac{x}{2} + 1\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin \frac{x}{2} = 0 \vee 2 \sin \frac{x}{2} + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin \frac{x}{2} = 0 \vee \sin \frac{x}{2} = -\frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \sin \frac{x}{2} = 0 \vee \sin \frac{x}{2} = \sin \left(-\frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{x}{2} = k\pi \vee \frac{x}{2} = -\frac{\pi}{6} + 2k\pi \vee \frac{x}{2} = \frac{7\pi}{6} + 2k\pi \Leftrightarrow$$

$$\Leftrightarrow x = 2k\pi \vee x = -\frac{\pi}{3} + 4k\pi \vee x = \frac{7\pi}{3} + 4k\pi, k \in \mathbb{Z}$$

Atribuindo valores a k obtemos as soluções do intervalo $[0, 4\pi]$:

$$x = 0 \vee x = 2\pi \vee x = 4\pi \vee x = -\frac{\pi}{3} + 4\pi \vee x = \frac{7\pi}{3} \Leftrightarrow$$

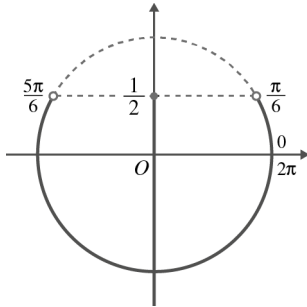
$$\Leftrightarrow x = 0 \vee x = 2\pi \vee x = \frac{7\pi}{3} \vee x = \frac{11\pi}{3} \vee x = 4\pi$$

18.2. $f(x) > \frac{1}{2} \Leftrightarrow 1 - \sin \frac{x}{2} > \frac{1}{2} \wedge x \in [0, 4\pi] \Leftrightarrow$

$$\Leftrightarrow \sin \frac{x}{2} < 1 - \frac{1}{2} \wedge \frac{x}{2} \in [0, 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin \frac{x}{2} < \frac{1}{2} \wedge \frac{x}{2} \in [0, 2\pi]$$

$$\sin \frac{x}{2} = \frac{1}{2} \wedge x \in [0, 2\pi] \Leftrightarrow \frac{x}{2} = \frac{\pi}{6} \vee \frac{x}{2} = \frac{5\pi}{6}$$



$$\sin \frac{x}{2} < \frac{1}{2} \wedge \frac{x}{2} \in [0, 2\pi] \Leftrightarrow \frac{x}{2} \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$$

$$\Leftrightarrow x \in \left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 4\pi\right]$$

18.3. $|g(x) - f(x)| = 1 \Leftrightarrow \left|3 - 2\cos^2 \frac{x}{2} - 1 + \sin \frac{x}{2}\right| = 1 \Leftrightarrow$

$$\Leftrightarrow \left|2 - 2\left(1 - \sin^2 \frac{x}{2}\right) + \sin \frac{x}{2}\right| = 1 \Leftrightarrow$$

$$\Leftrightarrow \left|2 - 2 + 2\sin^2 \frac{x}{2} + \sin \frac{x}{2}\right| = 1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 \frac{x}{2} + \sin \frac{x}{2} = 1 \vee 2\sin^2 \frac{x}{2} + \sin \frac{x}{2} = -1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 \frac{x}{2} + \sin \frac{x}{2} - 1 = 0 \vee 2\sin^2 \frac{x}{2} + \sin \frac{x}{2} + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2y^2 + y - 1 = 0 \vee 2y^2 + y + 1 = 0 \Leftrightarrow \left(y = \sin \frac{x}{2}\right)$$

$$\Leftrightarrow y = \frac{-1 \pm \sqrt{1+8}}{4} \vee y = \frac{-1 \pm \sqrt{1-8}}{4} \Leftrightarrow$$

$$\Leftrightarrow y = -1 \vee y = \frac{1}{2} \vee \text{condição impossível} \Leftrightarrow$$

$$\Leftrightarrow \sin \frac{x}{2} = -1 \vee \sin \frac{x}{2} = \frac{1}{2} \Leftrightarrow \sin \frac{x}{2} = -1 \vee \sin \frac{x}{2} = \sin \left(\frac{\pi}{6}\right)$$

$$\Leftrightarrow \frac{x}{2} = \frac{3\pi}{2} + 2k\pi \vee \frac{x}{2} = \frac{\pi}{6} + 2k\pi \vee$$

$$\vee \frac{x}{2} = \pi - \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = 3\pi + 4k\pi \vee x = \frac{\pi}{3} + 4k\pi \vee x = \frac{5\pi}{3} + 4k\pi, k \in \mathbb{Z}$$

No intervalo $[0, 4\pi]$, temos:

$$x = 3\pi \vee x = \frac{\pi}{3} \vee x = \frac{5\pi}{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{5\pi}{3} \vee x = 3\pi$$

$$\begin{cases} f\left(\frac{\pi}{3}\right) = 1 - \sin \frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2} & ; A\left(\frac{\pi}{3}, \frac{1}{2}\right) \\ g\left(\frac{\pi}{3}\right) = 3 - 2\cos^2 \frac{\pi}{6} = 3 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{2} & ; B\left(\frac{\pi}{3}, \frac{3}{2}\right) \end{cases}$$

$$\begin{cases} f\left(\frac{5\pi}{3}\right) = 1 - \sin \frac{5\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2} & ; A\left(\frac{5\pi}{3}, \frac{1}{2}\right) \\ g\left(\frac{5\pi}{3}\right) = 3 - 2\cos^2 \frac{5\pi}{6} = 3 - 2 \times \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{2} & ; B\left(\frac{5\pi}{3}, \frac{3}{2}\right) \end{cases}$$

$$\begin{cases} f(3\pi) = 1 - \sin \frac{3\pi}{2} = 1 + 1 = 2 & ; A(3\pi, 2) \\ g(3\pi) = 3 - 2\cos^2 \frac{3\pi}{2} = 3 - 0 = 3 & ; B(3\pi, 3) \end{cases}$$

$$A\left(\frac{\pi}{3}, \frac{1}{2}\right) \text{ e } B\left(\frac{\pi}{3}, \frac{3}{2}\right) ; A\left(\frac{5\pi}{3}, \frac{1}{2}\right) \text{ e } B\left(\frac{5\pi}{3}, \frac{3}{2}\right) \text{ ou } A(3\pi, 2) \text{ e } B(3\pi, 3)$$

Atividades complementares

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19. $f(x) = \frac{1 - \sin(4x)}{2}, D_f = \mathbb{R}$

19.1. $x \in \mathbb{R} \Leftrightarrow 4x \in \mathbb{R} \Leftrightarrow -1 \leq \sin(4x) \leq 1 \Leftrightarrow$

$$\Leftrightarrow 1 - 1 \leq 1 - \sin(4x) \leq 1 + 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{0}{2} \leq \frac{1 - \sin(4x)}{2} \leq \frac{2}{2} \Leftrightarrow 0 \leq f(x) \leq 1$$

$$D_f' = [0, 1]$$

19.2. Se $x \in D_f$ então $x + \frac{\pi}{2} \in D_f$ porque $D_f = \mathbb{R}$

$$\begin{aligned} f\left(x + \frac{\pi}{2}\right) &= \frac{1 - \sin\left[4\left(x + \frac{\pi}{2}\right)\right]}{2} = \\ &= \frac{1 - \sin(4x + 2\pi)}{2} = \\ &= \frac{1 - \sin(4x)}{2} = \\ &= f(x) \end{aligned}$$

$$f\left(x + \frac{\pi}{2}\right) = f(x), \forall x \in \mathbb{R}$$

19.3. $f(\pi + \alpha) \times f(\pi - \alpha) = \frac{4}{25} \Leftrightarrow$

$$\Leftrightarrow \frac{1 - \sin[4(\pi + \alpha)]}{2} \times \frac{1 - \sin[4(\pi - \alpha)]}{2} = \frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{1 - \sin(4\pi + 4\alpha)}{2} \times \frac{1 - \sin(4\pi - 4\alpha)}{2} = \frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{1 - \sin 4\alpha}{2} \times \frac{1 - \sin(-4\alpha)}{2} = \frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{(1 - \sin(4\alpha)) + (1 + \sin(4\alpha))}{4} = \frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow 1 - \sin^2(4\alpha) = \frac{16}{25} \Leftrightarrow \sin^2(4\alpha) = 1 - \frac{16}{25} \Leftrightarrow$$

$$\Leftrightarrow \sin(4\alpha) = \pm \sqrt{\frac{9}{25}}$$

Se $\alpha \in \left] \frac{\pi}{4}, \frac{\pi}{2} \right]$, $4\alpha \in]\pi, 2\pi[$.

Logo, $\sin(4\alpha) = -\frac{3}{5}$.

$$f(\alpha) = \frac{1 - \sin(4\alpha)}{2} = \frac{1 - \left(-\frac{3}{5}\right)}{2} = \frac{1 + \frac{3}{5}}{2} = \frac{\frac{8}{5}}{2} = \frac{4}{5}$$

20.1. Se $\widehat{BAP} = x$, então $\widehat{BOP} = 2x$.

Tomando $[OA]$ para base do triângulo $[AOP]$ a altura h é dada por $\sin(2x)$.

A área do triângulo $[AOP]$ é dada por:

$$\frac{\overline{AO} \times h}{2} = \frac{1 \times \sin(2x)}{2} = \frac{\sin(2x)}{2}$$

Como $\overline{OA} = \overline{OP}$, $[OS]$ é a mediana do triângulo $[AOP]$ relativa ao vértice O .

$$\text{Então, } A_{[OPS]} = \frac{1}{2} A_{[AOP]} = \frac{1}{2} \times \frac{\sin(2x)}{2} = \frac{1}{4} \sin(2x).$$

Portanto, $g(x) = \frac{1}{4} \sin(2x)$.

20.2. Se $x \in \left] 0, \frac{\pi}{2} \right]$, então $2x \in]0, \pi[$. Logo, $0 < \sin(2x) \leq 1$ e

$$0 < \frac{\sin(2x)}{4} \leq \frac{1}{4}. \text{ Assim, } D_g = \left] 0, \frac{1}{4} \right] \text{ e a área máxima do}$$

triângulo $[OPS]$

é $\frac{1}{4}$.

20.3. $P(\cos(2x), \sin(2x))$

$$\cos(2x) = \frac{5}{13}$$

$$\cos^2(2x) + \sin^2(2x) = 1$$

$$\left(\frac{5}{13}\right)^2 + \sin^2(2x) = 1 \Leftrightarrow \sin^2(2x) = 1 - \frac{25}{169} \Leftrightarrow$$

$$\Leftrightarrow \sin^2(2x) = \frac{144}{169}$$

Como $2x \in]0, \pi[$, temos $\sin(2x) = \frac{12}{13}$.

Se $\sin(2x) = \frac{12}{13}$, $g(x) = \frac{1}{4} \times \frac{12}{13} = \frac{3}{13}$.

21.1. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = x \Leftrightarrow -\frac{\sqrt{3}}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$$\Leftrightarrow x = -\frac{\pi}{3}$$

$$\arcsin \frac{1}{2} = x \Leftrightarrow \frac{1}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2\arcsin \frac{1}{2} = -\frac{\pi}{3} + 2 \times \frac{\pi}{6} = 0$$

21.2. $\sin\left(\frac{\pi}{2} + \arcsin \frac{\sqrt{2}}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) =$
 $= \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

22. $\cos\left(\arcsin \frac{3}{5}\right) = \cos x$

$$\arcsin \frac{3}{5} = x \Leftrightarrow \frac{3}{5} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \frac{9}{25} \Leftrightarrow \cos^2 x = \frac{16}{25}$$

Como $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos x > 0$. Logo, $\cos x = \frac{4}{5}$.

$$\cos\left(\arcsin \frac{3}{5}\right) = \cos x = \frac{4}{5}$$

23. $f(x) = 3\cos\left(\frac{x}{2}\right)$; $g(x) = \frac{1}{3}\sin(3x)$; $D_f = D_g = \mathbb{R}$

23.1. Se $x \in D_f$, então $x + 4\pi \in D_f$ porque $D_f = \mathbb{R}$.

$$f(x + 4\pi) = 3\cos\left(\frac{x + 4\pi}{2}\right) = 3\cos\left(\frac{x}{2} + \frac{4\pi}{2}\right) =$$

$$= 3\cos\left(\frac{x}{2} + 2\pi\right) = 3\cos \frac{x}{2} = f(x)$$

$$f(x + 4\pi) = f(x), \forall x \in \mathbb{R}$$

Logo, f é uma função periódica de período 4π .

23.2. Seja P o período positivo mínimo da função g .

Se $x \in D_g$, então $x + P \in D_g$ porque $D_g = \mathbb{R}$.

$$\forall x \in \mathbb{R}, g(x + P) = g(x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R}, \frac{1}{3}\sin[3(x + P)] = \frac{1}{3}\sin(3x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R}, \sin(3x + 3P) = \sin(3x)$$

Como 2π é o período positivo mínimo da função seno e P é o menor valor positivo para o qual a proposição é verdadeira, terá de ser $3P = 2\pi \Leftrightarrow P = \frac{2\pi}{3}$. Logo, a função g é periódica

de período positivo mínimo $P_0 = \frac{2\pi}{3}$.

23.3. $f(\pi - 2\alpha) = 1 \Leftrightarrow 3\cos\left(\frac{\pi - 2\alpha}{2}\right) = 1 \Leftrightarrow$

$$\Leftrightarrow 3\cos\left(\frac{\pi}{2} - \alpha\right) = 1 \Leftrightarrow 3\sin \alpha = 1 \Leftrightarrow \sin \alpha = \frac{1}{3}$$

$$g\left(\frac{\pi}{2} + \frac{\alpha}{3}\right) = \frac{1}{3}\sin\left[3\left(\frac{\pi}{2} + \frac{\alpha}{3}\right)\right] =$$

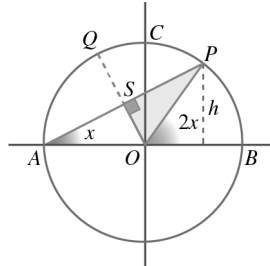
$$= \frac{1}{3}\sin\left(\frac{3\pi}{2} + \alpha\right) = \frac{1}{3}\sin\left(\pi + \frac{\pi}{2} + \alpha\right) =$$

$$= -\frac{1}{3}\sin\left(\frac{\pi}{2} + \alpha\right) = -\frac{1}{3}\cos \alpha = -\frac{1}{3} \times \left(-\frac{2\sqrt{2}}{3}\right) = \frac{2\sqrt{2}}{9}$$

Cálculo auxiliar:

$$\frac{1}{9} + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2 \alpha = \frac{8}{9}$$

Como $\alpha \in \left] \frac{\pi}{2}, \frac{3\pi}{2} \right]$, $\cos \alpha < 0$, pelo que $\cos \alpha = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$.



23.4. $D_h = D_f \cap D_g = \mathbb{R}$

Se $x \in D_h$, então $-x \in D_h$.

$$h(-x) = (f \times g)(-x) = f(-x) \times g(-x) =$$

$$= 3 \cos\left(\frac{-x}{2}\right) \times \frac{1}{3} \sin(3 \times (-x)) =$$

$$= 3 \cos\left(-\frac{x}{2}\right) \times \frac{1}{3} \sin(-3x) =$$

$$= 3 \cos \frac{x}{2} \times \frac{1}{3} [-\sin(3x)] =$$

$$= -3 \cos \frac{x}{2} \times \frac{1}{3} \sin(3x) =$$

$$= -(f \times g)(x) = -h(x)$$

$\forall x \in \mathbb{R}, h(-x) = -h(x)$

Logo, h é uma função ímpar.

24. $f(x) = \overline{BP}$; $g(x) = \overline{AP}$

24.1. O triângulo $[ABP]$ é retângulo em P porque o ângulo APB está inscrito numa semicircunferência.

24.2. Atendendo a que o triângulo $[ABP]$ é retângulo em P e a que

$$\widehat{BAP} = \frac{1}{2} \widehat{BOP} = \frac{x}{2}, \text{ temos:}$$

$$\frac{\overline{BP}}{\overline{AB}} = \sin\left(\frac{x}{2}\right) \Leftrightarrow \frac{\overline{BP}}{2} = \sin\left(\frac{x}{2}\right) \Leftrightarrow \overline{BP} = 2 \sin\left(\frac{x}{2}\right)$$

$$\frac{\overline{AP}}{\overline{AB}} = \cos\left(\frac{x}{2}\right) \Leftrightarrow \frac{\overline{AP}}{2} = \cos\left(\frac{x}{2}\right) \Leftrightarrow \overline{AP} = 2 \cos\left(\frac{x}{2}\right)$$

Portanto, $f(x) = 2 \sin\left(\frac{x}{2}\right)$ e $g(x) = 2 \cos\left(\frac{x}{2}\right)$.

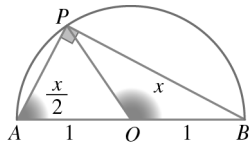
24.3. $f(\pi - 2\alpha) = 1 \Leftrightarrow 2 \sin\left(\frac{\pi - 2\alpha}{2}\right) = 1 \Leftrightarrow$

$$\Leftrightarrow 2 \sin\left(\frac{\pi}{2} - \alpha\right) = 1 \Leftrightarrow 2 \cos \alpha = 1 \Leftrightarrow$$

$$\Leftrightarrow \cos \alpha = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \alpha = \frac{\pi}{3} \quad \alpha \in]0, \pi[$$

$$\overline{AP} = g(\alpha) = g\left(\frac{\pi}{3}\right) = 2 \cos\left(\frac{\pi}{3}\right) = 2 \cos\left(\frac{\pi}{6}\right) = \sqrt{3}$$



25.2. $\cos\left(2 \arccos \frac{\sqrt{2}}{2} - \arccos \frac{\sqrt{3}}{2}\right) =$

$$= \cos\left(2 \times \frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

26. $\sin\left(\arccos \frac{12}{13}\right) = \sin x$

$$x = \arccos \frac{12}{13} \wedge x \in [0, \pi] \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{12}{13} \wedge x \in [0, \pi]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x + \left(\frac{12}{13}\right)^2 = 1 \Leftrightarrow \sin^2 x = 1 - \frac{144}{169} \Leftrightarrow \sin^2 x = \frac{25}{169}$$

Como $x \in [0, \pi]$, $\sin x > 0$. Logo, $\sin x = \frac{5}{13}$.

$$\sin\left(\arccos \frac{12}{13}\right) = \sin x = \frac{5}{13}$$

27. $h(x) = 2 \tan\left(\frac{x}{2} + \frac{\pi}{2}\right)$

27.1. $D_h = \mathbb{R} \setminus \left\{x: \frac{x}{2} + \frac{\pi}{2} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

$$= \mathbb{R} \setminus \{x: x = 2k\pi, k \in \mathbb{Z}\}$$

27.2. $h(\pi - x) \times h(x) = 2 \tan\left(\frac{\pi - x}{2} + \frac{\pi}{2}\right) \times 2 \tan\left(\frac{x}{2} + \frac{\pi}{2}\right) =$

$$= 4 \tan\left(\frac{\pi}{2} - \frac{x}{2} + \frac{\pi}{2}\right) \times \frac{\sin\left(\frac{\pi}{2} + \frac{x}{2}\right)}{\cos\left(\frac{\pi}{2} + \frac{x}{2}\right)} =$$

$$= 4 \tan\left(\pi - \frac{x}{2}\right) \times \frac{\cos\left(\frac{x}{2}\right)}{-\sin\left(\frac{x}{2}\right)} = -4 \tan\left(\frac{x}{2}\right) \times \frac{\cos\left(\frac{x}{2}\right)}{-\sin\left(\frac{x}{2}\right)} =$$

$$= 4 \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \times \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} = 4 \text{ se } \cos\left(\frac{x}{2}\right) \neq 0 \wedge \sin\left(\frac{x}{2}\right) \neq 0$$

$$\cos\left(\frac{x}{2}\right) \neq 0 \wedge \sin\left(\frac{x}{2}\right) \neq 0 \Leftrightarrow \frac{x}{2} \neq \frac{k\pi}{2}, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x \neq k\pi, k \in \mathbb{Z}$$

27.3. Se $x \in D_h$, então $x + 2\pi \in D_h$.

$$h(x + 2\pi) = 2 \tan\left(\frac{x + 2\pi}{2} + \frac{\pi}{2}\right) = 2 \tan\left(\frac{x}{2} + \pi + \frac{\pi}{2}\right) =$$

$$= 2 \tan\left[\left(\frac{x}{2} + \frac{\pi}{2}\right) + \pi\right] =$$

$$= 2 \tan\left(\frac{x}{2} + \frac{\pi}{2}\right) = h(x)$$

$$\forall x \in D_h, h(x + 2\pi) = h(x)$$

Logo, a função h é periódica de período 2π .

25.1. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = x \Leftrightarrow -\frac{\sqrt{3}}{2} = \cos x \wedge x \in [0, \pi]$

$$\Leftrightarrow x = \pi - \frac{\pi}{6} \Leftrightarrow x = \frac{5\pi}{6}$$

$$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) - \arccos\left(\frac{1}{2}\right) = \frac{5\pi}{6} - \frac{\pi}{3} = \frac{\pi}{2}$$

$$28.1. \arctan(-\sqrt{3}) = x \Leftrightarrow -\sqrt{3} = \tan x \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{3}$$

$$3\arctan\left(\frac{\sqrt{3}}{3}\right) + \arctan(-\sqrt{3}) = 3 \times \frac{\pi}{6} - \frac{\pi}{3} = \frac{\pi}{6}$$

$$28.2. \sin[\arctan(-1)] = \sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

Cálculo auxiliar:

$$\arctan(-1) = x \Leftrightarrow -1 = \tan x \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow x = -\frac{\pi}{4}$$

$$29. \tan\left(\arccos\frac{4}{5}\right) = \tan x \text{ com } \arccos\frac{4}{5} = x$$

$$\arccos\frac{4}{5} = x \Leftrightarrow \frac{4}{5} = \cos x \wedge x \in [0, \pi]$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \left(\frac{5}{4}\right)^2 \Leftrightarrow \tan^2 x = \frac{25}{16} - 1 \Leftrightarrow$$

$$\Leftrightarrow \tan^2 x = \frac{9}{16}$$

Como $x \in [0, \pi] \wedge \cos x > 0$, $\tan x > 0$.

$$\text{Logo, } \tan x = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

$$\text{Então, } \tan\left(\arccos\frac{4}{5}\right) = \tan x = \frac{3}{4}.$$

$$30.1. \sin\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(2x - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow 2x - \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \vee 2x - \frac{\pi}{3} = \pi - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{2\pi}{3} + 2k\pi \vee 2x = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{3} + k\pi \vee x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

$$30.2. \sin^3 x = \sin x \Leftrightarrow \sin^3 x - \sin x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x(\sin^2 x - 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \sin^2 x - 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee -\cos^2 x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \cos x = 0 \Leftrightarrow$$

$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = k\frac{\pi}{2}, k \in \mathbb{Z}$$

$$30.3. \sqrt{3} - \sin x = \sin x \Leftrightarrow$$

$$\Leftrightarrow \sin x + \sin x = \sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow 2\sin x = \sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow \sin x = \frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \sin x = \sin\frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = \pi - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$30.4. 1 - \sin x = \cos^2 x \Leftrightarrow$$

$$\Leftrightarrow 1 - \sin x = 1 - \sin^2 x \Leftrightarrow$$

$$\Leftrightarrow \sin^2 x - \sin x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x(\sin x - 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \sin x - 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \sin x = 1 \Leftrightarrow$$

$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$30.5. \sin x - 1 = \cos x - 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x = \cos x \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} - x + 2k\pi \vee x = \pi - \frac{\pi}{2} + x + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$30.6. 2\cos^2 x + 3\sin x - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2(1 - \sin^2 x) + 3\sin x - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2 - 2\sin^2 x + 3\sin x - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 x - 3\sin x + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2y^2 - 3y + 1 = 0 \Leftrightarrow \quad \left. \begin{array}{l} \\ \end{array} \right\} y = \sin x$$

$$\Leftrightarrow y = \frac{3 \pm \sqrt{9-8}}{4} \Leftrightarrow y = \frac{1}{2} \vee y = 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x = \frac{1}{2} \vee \sin x = 1 \Leftrightarrow \sin x = \sin\frac{\pi}{6} \vee \sin x = 1 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$31.1. (\sin x - 1)(2\sin x - 1) = 0 \wedge x \in]0, \pi[\Leftrightarrow$$

$$\Leftrightarrow (\sin x = 1 \vee 2\sin x - 1 = 0) \wedge x \in]0, \pi[\Leftrightarrow$$

$$\Leftrightarrow \left(\sin x = 1 \vee \sin x = \frac{1}{2}\right) \wedge x \in]0, \pi[\Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} \vee x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$$

$$S = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$$

$$31.2. 2\sin^2(2x) - \sin(2x) - 1 = 0 \wedge x \in \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow 2y^2 - y - 1 = 0 \wedge x \in \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \Leftrightarrow \quad \left. \begin{array}{l} \\ \end{array} \right\} y = \sin x$$

$$\Leftrightarrow y = \frac{1 \pm \sqrt{1+8}}{4} \wedge x \in \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow y = 1 \vee y = -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow \left(\sin(2x) = 1 \vee \sin(2x) = -\frac{1}{2}\right) \wedge 2x \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{2} \vee 2x = \pi + \frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{4} \vee x = \frac{7\pi}{12}$$

$$S = \left\{ \frac{\pi}{4}, \frac{7\pi}{12} \right\}$$

31.3. $\sin(2x) - \cos x = 0 \wedge x \in]-\pi, \pi[$

$$\sin(2x) - \cos x = 0 \Leftrightarrow \sin(2x) = \cos x \Leftrightarrow$$

$$\Leftrightarrow \sin(2x) = \sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{2} - x + 2k\pi \vee 2x = \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow 3x = \frac{\pi}{2} + 2k\pi \vee 2x = \pi - \frac{\pi}{2} + x + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3} \vee x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

Soluções no intervalo $]-\pi, \pi[$:

• $x = \frac{\pi}{6} + \frac{2k\pi}{3}$

$$k=0 \Rightarrow x = \frac{\pi}{6} \quad \checkmark$$

$$k=-1 \Rightarrow x = \frac{\pi}{6} - \frac{2\pi}{3} = -\frac{\pi}{2} \quad \checkmark$$

$$k=-2 \Rightarrow x = \frac{\pi}{6} - \frac{4\pi}{3} = -\frac{7\pi}{6} \quad \times$$

$$k=1 \Rightarrow x = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} \quad \checkmark$$

$$k=2 \Rightarrow x = \frac{\pi}{6} + \frac{4\pi}{3} = \frac{3\pi}{2} \quad \times$$

• $x = \frac{\pi}{2} + 2k\pi$

$$k=0 \Rightarrow x = \frac{\pi}{2} \quad \checkmark$$

$$k=-1 \Rightarrow x = \frac{\pi}{2} - 2\pi = -\frac{3\pi}{2} \quad \times$$

$$k=1 \Rightarrow x = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2} \quad \times$$

$$S = \left\{ -\frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$$

32.1. $\cos^2 x + \sin x(\sin x - 2\cos x) = 1 \Leftrightarrow$

$$\Leftrightarrow \cos^2 x + \sin^2 x - 2\sin x \cos x = 1 \Leftrightarrow$$

$$\Leftrightarrow 1 - 2\sin x \cos x = 1 \Leftrightarrow 2\sin x \cos x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin x = 0 \vee \cos x = 0 \Leftrightarrow$$

$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{2}, k \in \mathbb{Z}$$

32.2. $\cos(2x) - \sin x = 0 \Leftrightarrow \cos(2x) = \sin x \Leftrightarrow$

$$\Leftrightarrow \cos(2x) = \cos\left(\frac{\pi}{2} - x\right) \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{2} - x + 2k\pi \vee 2x = -\frac{\pi}{2} + x + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 3x = \frac{\pi}{2} + 2k\pi \vee x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3} \vee x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

32.3. $\sqrt{3}\sin^2 x = \sqrt{3} + 2\cos x \Leftrightarrow$

$$\Leftrightarrow \sqrt{3}(1 - \cos^2 x) = \sqrt{3} + 2\cos x \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3} - \sqrt{3}\cos^2 x - \sqrt{3} - 2\cos x = 0 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3}\cos^2 x + 2\cos x = 0 \Leftrightarrow \cos x(\sqrt{3}\cos x + 2) = 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = 0 \vee \cos x = -\frac{2}{\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \vee \text{condição impossível}$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

32.4. $3\sin^2 x = \cos^2 x \Leftrightarrow 3(1 - \cos^2 x) - \cos^2 x = 0 \Leftrightarrow$

$$\Leftrightarrow 3 - 3\cos^2 x - \cos^2 x = 0 \Leftrightarrow$$

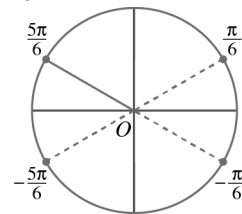
$$\Leftrightarrow 3 - 4\cos^2 x = 0 \Leftrightarrow \cos^2 x = \frac{3}{4} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{\sqrt{3}}{2} \vee \cos x = -\frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \cos x = \cos \frac{\pi}{6} \vee \cos x = \cos \left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee$$

$$x = -\frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$



$$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = -\frac{\pi}{6} + k\pi, k \in \mathbb{Z}$$

32.5. $\cos^3 x = \cos x \Leftrightarrow \cos^3 x - \cos x = 0 \Leftrightarrow$

$$\Leftrightarrow \cos x(\cos^2 x - 1) = 0 \Leftrightarrow \cos x = 0 \vee \cos^2 x = 1 \Leftrightarrow$$

$$\Leftrightarrow \cos x = 0 \vee \cos x = 1 \vee \cos x = -1 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi \vee x = 2k\pi \vee x = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{k\pi}{2}, k \in \mathbb{Z}$$

32.6. $\cos^2 x + \sin x + 1 = 0 \Leftrightarrow 1 - \sin^2 x + \sin x + 1 = 0 \Leftrightarrow$

$$\Leftrightarrow \sin^2 x - \sin x - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow y^2 - y - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow y = \frac{1 \pm \sqrt{1+8}}{2} \Leftrightarrow y = -1 \vee y = 2 \Leftrightarrow$$

$$\Leftrightarrow \sin x = -1 \vee \sin x = 2 \Leftrightarrow x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

33.1. $(1 - \cos x)(1 + 2\cos x) = 0 \wedge x \in]-\pi, \pi[\Leftrightarrow$

$$\Leftrightarrow (1 - \cos x = 0 \vee 1 + 2\cos x = 0) \wedge x \in]-\pi, \pi[\Leftrightarrow$$

$$\Leftrightarrow \left(\cos x = 1 \vee \cos x = -\frac{1}{2} \right) \wedge x \in]-\pi, \pi[\Leftrightarrow$$

$$\Leftrightarrow x = 0 \vee x = -\pi + \frac{\pi}{3} \vee x = \pi - \frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{2\pi}{3} \vee x = 0 \vee x = \frac{2\pi}{3}$$

$$S = \left\{ -\frac{2\pi}{3}, 0, \frac{2\pi}{3} \right\}$$

$$\begin{aligned}
 33.2. \quad & \sqrt{3} \sin x + 2 \sin x \cos x = 0 \wedge x \in \left] -\frac{\pi}{2}, \frac{3\pi}{2} \right[\Leftrightarrow \\
 & \Leftrightarrow \sin x (\sqrt{3} + 2 \cos x) = 0 \wedge x \in \left] -\frac{\pi}{2}, \frac{3\pi}{2} \right[\Leftrightarrow \\
 & \Leftrightarrow \sin x = 0 \vee \cos x = -\frac{\sqrt{3}}{2} \wedge x \in \left] -\frac{\pi}{2}, \frac{3\pi}{2} \right[\Leftrightarrow \\
 & \Leftrightarrow \sin x = 0 \vee \cos x = \cos \left(\pi - \frac{\pi}{6} \right) \wedge x \in \left] -\frac{\pi}{2}, \frac{3\pi}{2} \right[\Leftrightarrow \\
 & \Leftrightarrow \left(x = k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee x = -\frac{5\pi}{6} + 2k\pi \right) \wedge \\
 & \quad \wedge x \in \left] -\frac{\pi}{2}, \frac{3\pi}{2} \right[\Leftrightarrow \\
 & \Leftrightarrow x = 0 \vee x = \frac{5\pi}{6} \vee x = \pi \vee x = \frac{7\pi}{6}
 \end{aligned}$$

Cálculo auxiliar:

$$x = k\pi$$

$$k = -1 \Rightarrow x = -\pi \quad \times$$

$$k = 0 \Rightarrow x = 0 \quad \checkmark$$

$$k = 1 \Rightarrow x = \pi \quad \checkmark$$

$$k = 2 \Rightarrow x = 2\pi \quad \times$$

$$x = -\frac{5\pi}{6} + 2k\pi$$

$$k = 0 \Rightarrow x = -\frac{5\pi}{6} \quad \times$$

$$k = 1 \Rightarrow x = \frac{7\pi}{6} \quad \checkmark$$

$$k = 2 \Rightarrow x = \frac{19\pi}{6} \quad \times$$

$$x = \frac{5\pi}{6} + 2k\pi$$

$$k = 0 \Rightarrow x = \frac{5\pi}{6} \quad \checkmark$$

$$k = -1 \Rightarrow x = -\frac{7\pi}{6} \quad \times$$

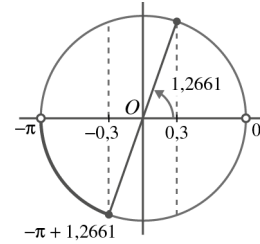
$$k = 1 \Rightarrow x = \frac{17\pi}{6} \quad \times$$

$$S = \left\{ 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6} \right\}$$

$$\begin{aligned}
 33.3. \quad & \sqrt{8} \cos\left(\frac{x}{2}\right) - 4 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) = 0 \wedge x \in]0, 2\pi[\Leftrightarrow \\
 & \Leftrightarrow \cos\left(\frac{x}{2}\right) (\sqrt{8} - 4 \sin\left(\frac{x}{2}\right)) = 0 \wedge \frac{x}{2} \in]0, \pi[\Leftrightarrow \\
 & \Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sqrt{8} - 4 \sin\left(\frac{x}{2}\right) = 0 \right) \wedge \frac{x}{2} \in]0, \pi[\Leftrightarrow \\
 & \Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sin\left(\frac{x}{2}\right) = \frac{\sqrt{8}}{4} \right) \wedge \frac{x}{2} \in]0, \pi[\Leftrightarrow \\
 & \Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sin\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2} \right) \wedge \frac{x}{2} \in]0, \pi[\Leftrightarrow \\
 & \Leftrightarrow \frac{x}{2} = \frac{\pi}{2} \vee \frac{x}{2} = \frac{\pi}{4} \vee \frac{x}{2} = \pi - \frac{\pi}{4} \Leftrightarrow \\
 & \Leftrightarrow x = \pi \vee x = \frac{\pi}{2} \vee x = \frac{3\pi}{2} \\
 & S = \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}
 \end{aligned}$$

$$\begin{aligned}
 34.1. \quad & 10 \cos(2x) + 3 = 0 \wedge x \in \left] -\frac{\pi}{2}, 0 \right[\Leftrightarrow \\
 & \Leftrightarrow \cos(2x) = -\frac{3}{10} \wedge 2x \in]-\pi, 0[
 \end{aligned}$$

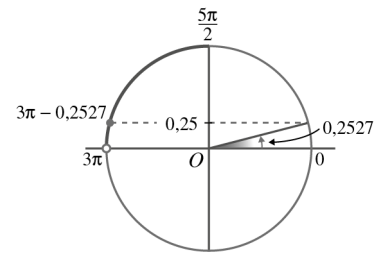
Utilizando a calculadora, obtemos $\arccos\left(\frac{3}{10}\right) \approx 1,2661$.



$$\begin{aligned}
 \cos(2x) &= -0,3 \wedge 2x \in]-\pi, 0[\Rightarrow \\
 \Rightarrow 2x &= (-\pi + 1,2661) \text{ rad} \Rightarrow \\
 \Rightarrow 2x &\approx -1,8755 \text{ rad} \Rightarrow x \approx -0,94 \text{ rad}
 \end{aligned}$$

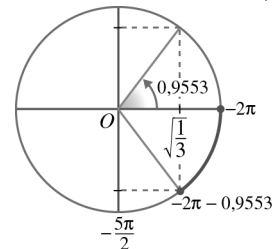
$$\begin{aligned}
 34.2. \quad & 2 \sin x = 0,5 \wedge x \in \left] \frac{5\pi}{2}, 3\pi \right[\Leftrightarrow \\
 & \Leftrightarrow \sin x = 0,25 \wedge x \in \left] \frac{5\pi}{2}, 3\pi \right[
 \end{aligned}$$

Com a calculadora obtemos: $\arcsin(0,25) \approx 0,2527$.



$$\begin{aligned}
 \sin x &= 0,25 \wedge x \in \left] \frac{5\pi}{2}, 3\pi \right[\Rightarrow \\
 \Rightarrow x &\approx (3\pi - 0,2527) \text{ rad} \Rightarrow x \approx 9,17 \text{ rad}
 \end{aligned}$$

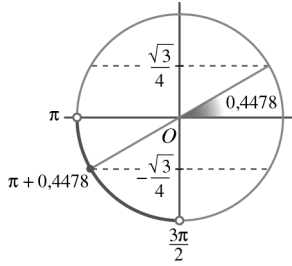
$$\begin{aligned}
 34.3. \quad & \cos \frac{x}{2} = \sqrt{\frac{1}{3}} \wedge x \in]-\pi, -4\pi[\Leftrightarrow \\
 & \Leftrightarrow \cos \frac{x}{2} = \sqrt{\frac{1}{3}} \wedge \frac{x}{2} \in \left] -\frac{5\pi}{2}, -2\pi \right[\\
 & \text{Na calculadora obtemos } \arccos\left(\sqrt{\frac{1}{3}}\right) \approx 0,9553.
 \end{aligned}$$



$$\begin{aligned}
 \cos\left(\frac{x}{2}\right) &= \sqrt{\frac{1}{3}} \wedge \frac{x}{2} \in \left] -\frac{5\pi}{2}, -2\pi \right[\Rightarrow \\
 \Rightarrow \frac{x}{2} &\approx (-2\pi - 0,9553) \text{ rad} \Rightarrow \\
 \Rightarrow \frac{x}{2} &\approx -7,2385 \text{ rad} \Rightarrow \\
 \Rightarrow x &\approx -14,48 \text{ rad}
 \end{aligned}$$

$$\begin{aligned}
 34.4. \quad & 4 \sin(4x) + \sqrt{3} = 0 \wedge x \in \left] \frac{\pi}{4}, \frac{3\pi}{8} \right[\Leftrightarrow \\
 & \Leftrightarrow \sin(4x) = -\frac{\sqrt{3}}{4} \wedge 4x \in \left] \pi, \frac{3\pi}{2} \right[
 \end{aligned}$$

Utilizando uma calculadora, $\arcsin\left(\frac{\sqrt{3}}{4}\right) \approx 0,4478$.



$$\sin(4x) = -\frac{\sqrt{3}}{4} \wedge 4x \in \left] \pi, \frac{3\pi}{2} \right[\Rightarrow$$

$$\Rightarrow 4x \approx (\pi + 0,4478) \text{ rad} \Rightarrow$$

$$\Rightarrow 4x \approx 3,5894 \text{ rad} \Rightarrow x \approx 0,90 \text{ rad}$$

35.1. $(\tan x + 1)\tan x = 0 \wedge x \in [0, \pi] \Leftrightarrow$

$$\Leftrightarrow (\tan x + 1 = 0 \vee \tan x = 0) \wedge x \in [0, \pi] \Leftrightarrow$$

$$\Leftrightarrow \tan x = -1 \vee \tan x = 0 \wedge x \in [0, \pi] \Leftrightarrow$$

$$\Leftrightarrow x = \pi - \frac{\pi}{4} \vee x = 0 \vee x = \pi \Leftrightarrow$$

$$\Leftrightarrow x = 0 \vee x = \frac{3\pi}{4} \vee x = \pi$$

$$S = \left\{ 0, \frac{3\pi}{4}, \pi \right\}$$

35.2. $3\tan\left(2x + \frac{\pi}{6}\right) - \sqrt{3} = 0 \wedge x \in]-\pi, 0[$

$$3\tan\left(2x + \frac{\pi}{6}\right) - \sqrt{3} = 0 \Leftrightarrow \tan\left(2x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow 2x + \frac{\pi}{6} = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{2}, k \in \mathbb{Z}$$

Para $k = -1$, $x = -\frac{\pi}{2}$ única solução do intervalo $]-\pi, 0[$.

$$S = \left\{ -\frac{\pi}{2} \right\}$$

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36.1. $\tan\left(\frac{3x}{2}\right) + 1 = 0 \Leftrightarrow \tan\left(\frac{3x}{2}\right) = -1 \Leftrightarrow$

$$\Leftrightarrow \tan\left(\frac{3x}{2}\right) = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{3x}{2} = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 3x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{6} + \frac{2k\pi}{3}, k \in \mathbb{Z}$$

36.2. $3 - \tan^2(2x) = 0 \Leftrightarrow$

$$\Leftrightarrow \tan^2(2x) = 3 \Leftrightarrow$$

$$\Leftrightarrow \tan(2x) = \sqrt{3} \vee \tan(2x) = -\sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow \tan(2x) = \tan\left(\frac{\pi}{3}\right) \vee \tan(2x) = \tan\left(-\frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{3} + k\pi \vee 2x = -\frac{\pi}{3} + k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{2} \vee x = -\frac{\pi}{6} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

36.3. $\tan^3 x = \tan x \Leftrightarrow$

$$\Leftrightarrow \tan^3 x - \tan x = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan x(\tan^2 x - 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan x = 0 \vee \tan^2 x - 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan x = 0 \vee \tan x = 1 \vee \tan x = -1 \Leftrightarrow$$

$$\Leftrightarrow \tan x = 0 \vee \tan x = \tan\frac{\pi}{4} \vee \tan x = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

36.4. $2\tan x - \tan x \sin^2 x = 0 \Leftrightarrow$

$$\Leftrightarrow \tan x(2 - \sin^2 x) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tan x = 0 \vee \sin^2 x = 2 \Leftrightarrow$$

$$\Leftrightarrow x = k\pi \vee \text{condição impossível}, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = k\pi, k \in \mathbb{Z}$$

36.5. $\cos x + \sin x \tan x = 2 \Leftrightarrow$

$$\Leftrightarrow \cos x + \sin x \times \frac{\sin x}{\cos x} - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow \cos^2 x + \sin^2 x - 2\cos x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow 1 - 2\cos x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

36.6. $\tan^2 x + 1 = 2\tan x \Leftrightarrow$

$$\Leftrightarrow \tan^2 x - 2\tan x + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow y^2 - 2y + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow y = \frac{2 \pm \sqrt{4-4}}{2} \Leftrightarrow y = 1 \Leftrightarrow$$

$$\Leftrightarrow \tan x = 1 \Leftrightarrow \tan x = \tan\frac{\pi}{4} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$y = \tan x$

36.7. $\tan^2 x + 1 = \frac{2}{\cos x} \Leftrightarrow$

$$\Leftrightarrow \frac{\sin^2 x}{\cos^2 x} - \frac{2}{\cos x} + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sin^2 x - 2\cos x + \cos^2 x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow 1 - 2\cos x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{1}{2} \wedge \cos x \neq 0 \Leftrightarrow \cos x = \cos\frac{\pi}{3} \Leftrightarrow$$

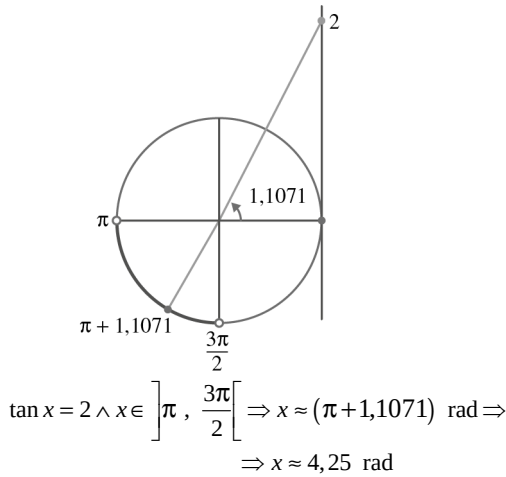
$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

37. $2\cos^2 x = \tan x - 2\sin^2 x \wedge x \in \left] \pi, \frac{3\pi}{2} \right[\Leftrightarrow$

$$\Leftrightarrow 2(\cos^2 x + \sin^2 x) = \tan x \wedge x \in \left] \pi, \frac{3\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \tan x = 2 \wedge x \in \left] \pi, \frac{3\pi}{2} \right[$$

Recorrendo à calculadora, $\arctan(2) \approx 1,1071$.



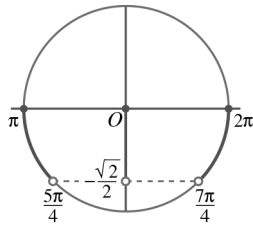
38.1. $6\sin x + \sqrt{18} > 0 \wedge x \in [\pi, 2\pi] \Leftrightarrow$

$$\Leftrightarrow \sin x > -\frac{3\sqrt{2}}{6} \wedge x \in [\pi, 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin x > -\frac{\sqrt{2}}{2} \wedge x \in [\pi, 2\pi]$$

• $\sin x = -\frac{\sqrt{2}}{2} \wedge x \in [\pi, 2\pi] \Leftrightarrow$

$$\Leftrightarrow x = \pi + \frac{\pi}{4} \vee x = 2\pi - \frac{\pi}{4} \Leftrightarrow x = \frac{5\pi}{4} \vee x = \frac{7\pi}{4}$$



• $\sin x > -\frac{\sqrt{2}}{2} \wedge x \in [\pi, 2\pi] \Leftrightarrow$

$$\Leftrightarrow x \in \left[\pi, \frac{5\pi}{4} \right[\cup \left] \frac{7\pi}{4}, 2\pi \right]$$

38.2. $\sqrt{12}\cos x + \sqrt{3} > 0 \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right] \Leftrightarrow$

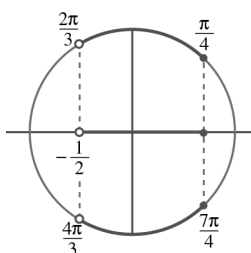
$$\Leftrightarrow \cos x > -\frac{\sqrt{3}}{\sqrt{12}} \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow \cos x > -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow \cos x > -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right]$$

• $\cos x = -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right] \Leftrightarrow$

$$\Leftrightarrow x = \pi - \frac{\pi}{3} \vee x = \pi + \frac{\pi}{3} \Leftrightarrow x = \frac{2\pi}{3} \vee x = \frac{4\pi}{3}$$



$$\cos x > -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right] \Leftrightarrow$$

$$\Leftrightarrow x \in \left[\frac{\pi}{4}, \frac{2\pi}{3} \right[\cup \left] \frac{4\pi}{3}, \frac{7\pi}{4} \right]$$

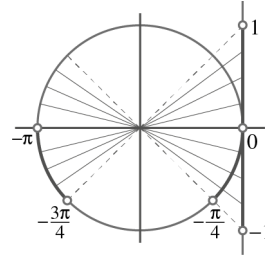
38.3. $|\tan x| < 1 \wedge x \in]-\pi, 0[\Leftrightarrow$

$$\Leftrightarrow \tan x > -1 \wedge \tan x < 1 \wedge x \in]-\pi, 0[\Leftrightarrow$$

$$\Leftrightarrow -1 < \tan x < 1 \wedge x \in]-\pi, 0[$$

$$(\tan x = -1 \vee \tan x = 1) \wedge x \in]-\pi, 0[\Leftrightarrow$$

$$\Leftrightarrow x = -\pi + \frac{\pi}{4} \vee x = -\frac{\pi}{4} \Leftrightarrow x = -\frac{3\pi}{4} \vee x = -\frac{\pi}{4}$$



$$-1 < \tan x < 1 \wedge x \in]-\pi, 0[\Leftrightarrow$$

$$\Leftrightarrow x \in \left] -\pi, -\frac{3\pi}{4} \right[\cup \left] -\frac{\pi}{4}, 0 \right[$$

39. $f(x) = 1 - 2\cos(2\pi x)$

$$g(x) = \sin(\pi x + a), a \in \mathbb{R}$$

$$D_f = D_g = [0, 2]$$

39.1. $x \in [0, 2] \Leftrightarrow 2\pi x \in [0, 4\pi]$

$$-1 \leq \cos(2\pi x) \leq 1$$

$$-2 \leq -2\cos(2\pi x) \leq 2$$

$$1 - 2 \leq 1 - 2\cos(2\pi x) \leq 1 + 2$$

$$-1 \leq f(x) \leq 3$$

$$D'_f = [-1, 3]$$

39.2. $f(1) = g(1) \Leftrightarrow$

$$\Leftrightarrow 1 - 2\cos(2\pi) = \sin(\pi + a) \Leftrightarrow$$

$$\Leftrightarrow 1 - 2 = -\sin a \Leftrightarrow \sin a = 1 \Leftrightarrow$$

$$\Leftrightarrow a = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

O menor valor positivo de a é $\frac{\pi}{2}$.

39.3. Seja A o ponto de menor abscissa tal que $\overline{AB} = \frac{3}{4}$.

Então, B tem abscissa $x + \frac{3}{4}$ e $f(x) = f\left(x + \frac{3}{4}\right)$.

$$f(x) = f\left(x + \frac{3}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow 1 - 2\cos(2\pi x) = 1 - 2\cos\left[2\pi\left(x + \frac{3}{4}\right)\right] \Leftrightarrow$$

$$\Leftrightarrow \cos(2\pi x) = \cos\left(2\pi x + \frac{3\pi}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow 2\pi x = 2\pi x + \frac{3\pi}{2} + 2k\pi \vee 2\pi x =$$

$$= -2\pi x - \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \text{condição impossível} \vee 4\pi x = -\frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow 4x = -\frac{3}{2} + 2k, k \in \mathbb{Z} \Leftrightarrow x = -\frac{3}{8} + \frac{k}{2}, k \in \mathbb{Z}$$

Como $x \in [0, 2]$ e $x + \frac{3}{4} \in [0, 2]$, temos:

$$k=0 \Rightarrow x = -\frac{3}{8} \notin D_f$$

$$k=1 \Rightarrow x = \frac{1}{8} \text{ e } x + \frac{3}{4} = \frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$

$$k=2 \Rightarrow x = \frac{5}{8} \text{ e } x + \frac{3}{4} = \frac{5}{8} + \frac{3}{4} = \frac{11}{8}$$

$$k=3 \Rightarrow x = \frac{9}{8} \text{ e } x + \frac{3}{4} = \frac{9}{8} + \frac{3}{4} = \frac{15}{8}$$

$$k=4 \Rightarrow x = \frac{13}{8} \text{ e } x + \frac{3}{4} = \frac{13}{8} + \frac{3}{4} = \frac{19}{8} \notin D_f$$

$$f\left(\frac{1}{8}\right) = 1 - 2\cos\frac{\pi}{4} = 1 - \sqrt{2} = f\left(\frac{7}{8}\right)$$

$$f\left(\frac{5}{8}\right) = 1 - 2\cos\frac{5\pi}{4} = 1 + \sqrt{2} = f\left(\frac{11}{8}\right)$$

$$f\left(\frac{9}{8}\right) = 1 - 2\cos\frac{9\pi}{4} = 1 - \sqrt{2} = f\left(\frac{15}{8}\right)$$

Os pares de pontos (A, B) que verificam a condição são

$$A\left(\frac{1}{8}, 1 - \sqrt{2}\right) \text{ e } B\left(\frac{7}{8}, 1 - \sqrt{2}\right); A\left(\frac{5}{8}, 1 + \sqrt{2}\right) \text{ e }$$

$$B\left(\frac{11}{8}, 1 + \sqrt{2}\right) \text{ ou } A\left(\frac{9}{8}, 1 - \sqrt{2}\right) \text{ e } B\left(\frac{15}{8}, 1 - \sqrt{2}\right)$$

40.1. $[OP] \perp [OA]$

$$\frac{\overline{OP}}{\overline{OA}} = \cos \alpha \Leftrightarrow \frac{\overline{OP}}{r} = \cos \alpha \Leftrightarrow \overline{OP} = r \cos \alpha$$

$$\frac{\overline{PA}}{\overline{OA}} = \sin \alpha \Leftrightarrow \frac{\overline{PA}}{r} = \sin \alpha \Leftrightarrow \overline{PA} = r \sin \alpha$$

$$A_{[OAP]} = \frac{\overline{PA} \times \overline{OP}}{2} = \frac{r \sin \alpha \times r \cos \alpha}{2} = \frac{r^2}{2} \sin \alpha \cos \alpha$$

$$40.2. 3\overline{AP} = \sqrt{3}\overline{OP} \Leftrightarrow \overline{AP} = \frac{\sqrt{3}}{3}\overline{OP} \Leftrightarrow$$

$$\Leftrightarrow \frac{\overline{AP}}{\overline{OP}} = \frac{\sqrt{3}}{3} \Leftrightarrow \tan \alpha = \frac{\sqrt{3}}{3}$$

$$\text{Se } \tan \alpha = \frac{\sqrt{3}}{3} \wedge \alpha \in \left]0, \frac{\pi}{2}\right[, \text{ então } \alpha = \frac{\pi}{6} \text{ e } \cos \alpha = \frac{\sqrt{3}}{2}.$$

$$\overline{OP} = r \cos \alpha = \frac{\sqrt{3}}{2}r$$

$$40.3. \overline{OP} = \frac{r}{2} \Leftrightarrow r \cos \alpha = \frac{r}{2} \Leftrightarrow \cos \alpha = \frac{1}{2}$$

$$\text{Como } \alpha \in \left]0, \frac{\pi}{2}\right[, \text{ temos que } \alpha = \frac{\pi}{3}.$$

$$A_{[OAP]} = \frac{r^2}{2} \sin \alpha \cos \alpha =$$

$$= \frac{r^2}{2} \times \sin \frac{\pi}{3} \times \cos \frac{\pi}{3} =$$

$$= \frac{r^2}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{8}r^2$$

$$40.4. r = 1; P(a, b) \text{ e } a = \frac{3}{4}$$

$$\frac{a}{\overline{OP}} = \cos \alpha$$

$$a = \overline{OP} \times \cos \alpha$$

$$\frac{3}{4} = r \cos \alpha \times \cos \alpha$$

$$\frac{3}{4} = \cos^2 \alpha$$

$$\cos \alpha = \pm \frac{\sqrt{3}}{2}$$

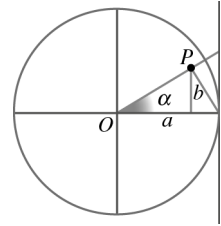
$$\text{Como } \alpha \in 1.^\circ Q, \cos \alpha = \frac{\sqrt{3}}{2} \text{ e } \alpha = \frac{\pi}{6}.$$

$$\frac{b}{\overline{OP}} = \sin \alpha \Leftrightarrow b = \overline{OP} \times \sin \alpha \Leftrightarrow$$

$$\Leftrightarrow b = r \cos \alpha \times \sin \alpha$$

$$\text{Para } r = 1 \text{ e } \alpha = \frac{\pi}{6}, \text{ vem:}$$

$$b = 1 \times \sin \frac{\pi}{6} \times \cos \frac{\pi}{6} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$



$$40.5. A_{[OAQ]} = \frac{\overline{OA} \times \overline{AQ}}{2}$$

$$\frac{\overline{AQ}}{\overline{OA}} = \tan \alpha \Leftrightarrow \frac{\overline{AQ}}{r} = \tan \alpha \Leftrightarrow \overline{AQ} = r \tan \alpha$$

$$A_{[OAQ]} = \frac{r \times r \tan \alpha}{2} = \frac{r^2}{2} \tan \alpha$$

$$A_{[AQP]} = A_{[OAQ]} - A_{[OAP]} = \frac{r^2}{2} \tan \alpha - \frac{r^2}{2} \sin \alpha \cos \alpha =$$

$$= \frac{r^2}{2} (\tan \alpha - \sin \alpha \cos \alpha) =$$

$$= \frac{r^2}{2} \left(\frac{\sin \alpha}{\cos \alpha} - \sin \alpha \cos \alpha \right) =$$

$$= \frac{r^2}{2} \frac{\sin \alpha - \sin \alpha \cos^2 \alpha}{\cos \alpha} = \frac{r^2 \sin \alpha (1 - \cos^2 \alpha)}{2 \cos \alpha} =$$

$$= \frac{r^2 \sin \alpha \times \sin^2 \alpha}{2 \cos \alpha} = \frac{r^2 \sin^3 \alpha}{2 \cos \alpha}$$

$$40.6. r = 1; \overline{AQ} = \sqrt{8}$$

$$r \tan \alpha = \sqrt{8}$$

$$\tan \alpha = \sqrt{8}$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + (\sqrt{8})^2 = \frac{1}{\cos^2 \alpha} \Leftrightarrow 1 + 8 = \frac{1}{\cos^2 \alpha} \Leftrightarrow \cos^2 \alpha = \frac{1}{9}$$

$$\text{Como } \alpha \in 1.^\circ Q, \text{ vem } \cos \alpha = \frac{1}{3}.$$

$$\sin^2 \alpha + \frac{1}{9} = 1 \Leftrightarrow \sin^2 \alpha = \frac{8}{9}$$

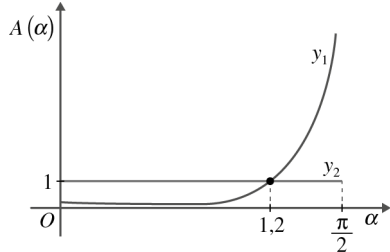
$$\text{Como } \alpha \in 1.^\circ Q, \sin \alpha = \frac{\sqrt{8}}{3}.$$

$$A = \frac{1 \times \left(\frac{\sqrt{8}}{3}\right)^3}{2 \times \frac{1}{3}} = \frac{\frac{8 \times \sqrt{8}}{27}}{\frac{2}{3}} = \frac{8 \times 2\sqrt{2} \times 3}{27 \times 2} = \frac{8\sqrt{2}}{9}$$

40.7. Se $r=1$, $A(\alpha) = \frac{\sin^3 \alpha}{2 \cos \alpha}$.

Fazendo $y_1 = \frac{\sin^3 x}{2 \cos x}$ e $y_2 = 1$, determinou-se, no intervalo

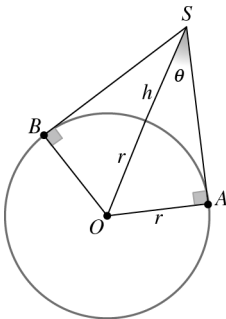
$\left[0, \frac{\pi}{2}\right]$ a abscissa do ponto de interseção dos dois gráficos:



Logo, $\alpha \approx 1,2$.

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41.



41.1. $\frac{r}{h+r} = \sin \theta \Leftrightarrow r = h \sin \theta + r \sin \theta \Leftrightarrow$
 $\Leftrightarrow h \sin \theta = r - r \sin \theta \Leftrightarrow h = \frac{r(1 - \sin \theta)}{\sin \theta}$
 Como $r = 6370$ km: $h = \frac{6370(1 - \sin \theta)}{\sin \theta}$

41.2. $d =$ comprimento do arco AB

$$\frac{2\pi R}{d} = \frac{2\pi}{A\hat{O}B}$$

$$d = r \times A\hat{O}B$$

$$A\hat{O}B = 2 \times A\hat{O}S = 2 \left(\frac{\pi}{2} - \theta \right) = \pi - 2\theta$$

$$d = r(\pi - 2\theta)$$

$$d = 6370(\pi - 2\theta)$$

41.3. $h = 150$ km

$$150 = \frac{6370(1 - \sin \theta)}{\sin \theta} \Leftrightarrow$$

$$\Leftrightarrow 150 \sin \theta = 6370 - 6370 \sin \theta \Leftrightarrow$$

$$\Leftrightarrow 150 \sin \theta + 6370 \sin \theta = 6370 \Leftrightarrow$$

$$\Leftrightarrow 6520 \sin \theta = 6370 \Leftrightarrow$$

$$\Leftrightarrow \sin \theta = \frac{6370}{6520} \Leftrightarrow$$

$$\Leftrightarrow \theta = \arcsin \left(\frac{637}{652} \right)$$

$$d = 6370(\pi - 2\theta) =$$

$$= 6370 \left(\pi - 2 \arcsin \frac{637}{652} \right) \approx 2738 \text{ km}$$

41.4. $d = 1450$ km

$$d = 6370(\pi - 2\theta)$$

$$1450 = 6370\pi - 12740\theta$$

$$\theta = \frac{6370\pi - 1450}{12740}$$

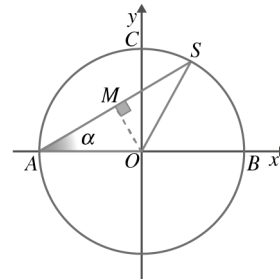
$$\theta \approx 1,45698$$

$$h = \frac{6370(1 - \sin \theta)}{\sin \theta}$$

$$h \approx \frac{6370(1 - \sin(1,45698))}{\sin(1,45698)}$$

$$h \approx 41,5 \text{ km}$$

42.



42.1. $\overline{AO} = \overline{OS} = r$

Seja M o ponto médio de $[AS]$

O triângulo $[AOM]$ é retângulo em M .

$$\frac{\overline{AM}}{\overline{AO}} = \cos \alpha \Leftrightarrow \frac{\overline{AM}}{r} = \cos \alpha \Leftrightarrow \overline{AM} = r \cos \alpha$$

$$\overline{AS} = 2\overline{AM} = 2r \cos \alpha$$

$$P_{[AOS]} = \overline{AO} + \overline{OS} + \overline{AS} = r + r + 2r \cos \alpha =$$

$$= 2r + 2r \cos \alpha$$

$$P_{[AOS]} = 2r(1 + \cos \alpha)$$

42.2. $r = 1$

$$\widehat{BS} = 2\widehat{SA} \Leftrightarrow \widehat{SA} = \frac{\widehat{BS}}{2}$$

$$\widehat{BS} + \widehat{SA} = \pi \Leftrightarrow \widehat{BS} + \frac{\widehat{BS}}{2} = \pi \Leftrightarrow$$

$$\Leftrightarrow 2\widehat{BS} + \widehat{BS} = 2\pi \Leftrightarrow \widehat{BS} = \frac{2\pi}{3}$$

$$\alpha = \frac{1}{2}\widehat{BS} = \frac{1}{2} \times \frac{2\pi}{3} = \frac{\pi}{3}$$

$$P_{[AOS]} = 2 \times 1 \left(1 + \cos \frac{\pi}{3} \right) = 2 \left(1 + \frac{1}{2} \right) = 3$$

42.3. $P = 3r \Leftrightarrow$

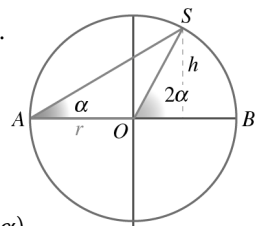
$$2r(1 + \cos \alpha) = 3r \Leftrightarrow 1 + \cos \alpha = \frac{3}{2} \Leftrightarrow \cos \alpha = \frac{1}{2}$$

Como $\alpha \in \left[0, \frac{\pi}{2}\right]$, temos $\alpha = \frac{\pi}{3}$.

Tomando $[AO]$ para base do triângulo $[AOS]$, a sua altura, h , é:

$$\frac{h}{\overline{OS}} = \sin(2\alpha) \Leftrightarrow h = r \sin(2\alpha)$$

$$A_{[AOS]} = \frac{\overline{AO} \times r \sin(2\alpha)}{2} = \frac{r^2 \sin(2\alpha)}{2}$$



Para $\alpha = \frac{\pi}{3}$:

$$A_{[AOS]} = \frac{r^2 \times \sin\left(\frac{2\pi}{3}\right)}{2} = \frac{r^2 \times \sin\left(\pi - \frac{\pi}{3}\right)}{2}$$

$$= \frac{1}{2} \times r^2 \times \sin \frac{\pi}{3} = \frac{r^2}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} r^2$$

42.4. $r = 1$; $P = 2 + \sqrt{3}$

$$P = 2r(1 + \cos \alpha)$$

$$2 + \sqrt{3} = 2(1 + \cos \alpha) \Leftrightarrow 2 + \sqrt{3} = 2 + 2 \cos \alpha \Leftrightarrow$$

$$\Leftrightarrow 2 \cos \alpha = \sqrt{3} \Leftrightarrow \alpha = \frac{\sqrt{3}}{2}$$

Como $\alpha \in \left]0, \frac{\pi}{2}\right[$, vem $\alpha = \frac{\pi}{6}$.

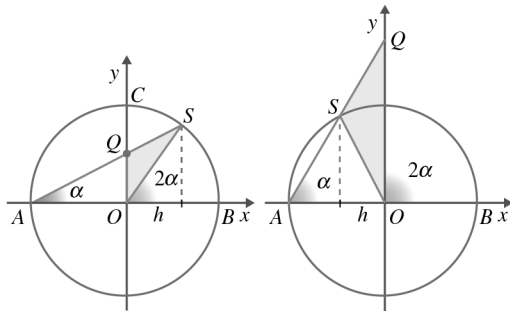
Seja C o comprimento do arco BS .

O ângulo ao centro correspondente ao arco BS tem amplitude

$$2\alpha = 2 \times \frac{\pi}{6} = \frac{\pi}{3}.$$

$$C = r \times \frac{\pi}{3} = 1 \times \frac{\pi}{3} = \frac{\pi}{3}$$

42.5.



a) Tomando $[OQ]$ para base do triângulo $[OSQ]$, a sua altura h é dada por:

$$\frac{h}{r} = \cos 2\alpha \Leftrightarrow h = r \cos 2\alpha, \text{ se } 2\alpha \leq \frac{\pi}{2}$$

$$\frac{h}{r} = \cos(\pi - 2\alpha) \Leftrightarrow h = -r \cos(2\alpha), \text{ se } 2\alpha > \frac{\pi}{2}$$

Portanto, $h = r |\cos(2\alpha)|$.

$$\frac{\overline{OQ}}{\overline{AO}} = \tan \alpha \Leftrightarrow \overline{OQ} = r \tan \alpha$$

$$A_{[OSQ]} = \frac{\overline{OQ} \times h}{2} = \frac{1}{2} \times r \tan \alpha \times r |\cos(2\alpha)|$$

$$A(\alpha) = \frac{1}{2} r^2 \tan \alpha |\cos(2\alpha)|$$

b) $A(\alpha) = 0 \Leftrightarrow \frac{1}{2} r^2 \tan \alpha |\cos(2\alpha)| = 0 \Leftrightarrow$

$$\Leftrightarrow \tan \alpha = 0 \vee |\cos(2\alpha)| = 0 \Leftrightarrow$$

$$\Leftrightarrow \cos(2\alpha) = 0 \text{ pois } \tan \alpha \neq 0, \forall \alpha \in \left]0, \frac{\pi}{2}\right[$$

$$\Leftrightarrow 2\alpha = \frac{\pi}{2} \Leftrightarrow$$

$$\Leftrightarrow \alpha = \frac{\pi}{4}$$

Se $\alpha = \frac{\pi}{4}$, $[OSQ]$ reduz-se ao segmento de reta $[OC]$.

c) $r = 3$

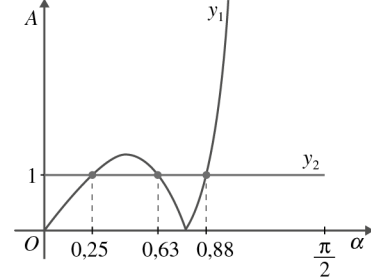
$$A(\alpha) = \frac{9}{2} \tan \alpha |\cos(2\alpha)|$$

Considerando na calculadora gráfica:

$$y_1 = \frac{9}{2} \tan x |\cos(2x)| \quad \text{e} \quad y_2 = 1$$

com $x \in \left]0, \frac{\pi}{2}\right[$, determinam-se as abscissas dos pontos

de interseção dos dois gráficos:



A área do triângulo $[OSQ]$ é igual a 1 para, $\alpha \approx 0,25 \vee \alpha \approx 0,63 \vee \alpha \approx 0,88$

Avaliação 3

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1. Como $P_0 < 5$, da análise do gráfico, podemos concluir que $P_0 = 3$.

$$f(x + kP) = f(x), \forall x \in D_f$$

• $f(2) = f(2 + 6 \times 3) = f(20)$ ((A) é verdadeira)

• $f(13) = f(13 + 4 \times 3) = f(25)$

$$f(25) = f(13) \Leftrightarrow f(25) - f(13) = 0 \quad ((B) \text{ é verdadeira})$$

• $f(12) = f(0 + 4 \times 3) = f(0) \neq f(4)$ ((C) é falsa)

• $f(8) = f(2 + 2 \times 3) = f(2)$

$$f(15) = f(6 + 3 \times 3) = f(6)$$

$$\text{Logo, } f(8) + f(6) = f(15) + f(2) \quad ((D) \text{ é verdadeira})$$

Resposta: (C)

2. $f(x) = \cos(\pi x)$; $\overline{AB} = \frac{1}{2}$

Se x é abscissa de A, $x + \frac{1}{2}$ é a abscissa de B.

$$f(x) = f\left(x + \frac{1}{2}\right)$$

$$\cos(\pi x) = \cos\left(\pi\left(x + \frac{1}{2}\right)\right) \Leftrightarrow \cos(\pi x) = \cos\left(\pi x + \frac{\pi}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow \pi x = \pi x + \frac{\pi}{2} + 2k\pi \vee \pi x = -\pi x - \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow \text{condição impossível} \vee 2x = -\frac{1}{2} + 2k, k \in \mathbb{Z} \Leftrightarrow$$

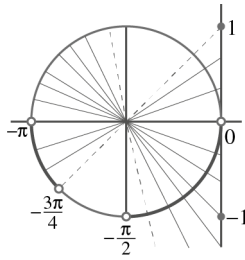
$$\Leftrightarrow x = -\frac{1}{4} + k, k \in \mathbb{Z}$$

Para $k = 1$, temos: $x = \frac{3}{4}$ e $x + \frac{1}{2} = \frac{5}{4}$

Resposta: (C)

3. $\tan x \leq 1 \wedge x \in]-\pi, 0[$

No intervalo $]-\pi, 0[$, $\tan x = 1 \Leftrightarrow x = -\frac{3\pi}{4}$.



$\tan x \leq 1 \wedge x \in]-\pi, 0[\Leftrightarrow$

$\Leftrightarrow x \in]-\pi, -\frac{3\pi}{4}] \cup]-\frac{\pi}{2}, 0[$

Resposta: (A)

4. $f(x) = \sin x + \cos x$

4.1. $f(-x) = \sin(-x) + \cos(-x) =$
 $= -\sin x + \cos x$

A função f não é par nem ímpar.

$\forall x \in \mathbb{R}, f\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) =$
 $= \cos x - \sin x =$
 $= \cos(-x) + \sin(-x) =$
 $= f(-x)$

Resposta: (D)

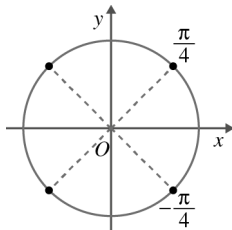
4.2. $g(x) = f(-x) + f(x) =$
 $= [\sin(-x) + \cos(-x)] + (\sin x + \cos x) =$
 $= (-\sin x + \cos x) + (\sin x + \cos x) =$
 $= (\cos x - \sin x) + (\cos x + \sin x) =$
 $= \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x =$
 $= 1 - 2\sin^2 x$

Resposta: (D)

5. $|\tan x| - 1 = 0 \Leftrightarrow |\tan x| = 1 \Leftrightarrow \tan x = 1 \vee \tan x = -1 \Leftrightarrow$

$\Leftrightarrow \tan x = \tan \frac{\pi}{4} \vee \tan x = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \Leftrightarrow$



$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} - \frac{k\pi}{2}, k \in \mathbb{Z}$

Resposta: (A)

6. $h(x) = 2\sin(4\pi x + a), a \in \mathbb{R}; D_h = \mathbb{R}$

6.1. Se $x \in D_h$, então $x + \frac{1}{2} \in D_h$ porque $D_h = \mathbb{R}$.

$h\left(x + \frac{1}{2}\right) = 2\sin\left[4\pi\left(x + \frac{1}{2}\right) + a\right] =$
 $= 2\sin(4\pi x + 2\pi + a) =$
 $= 2\sin[(4\pi x + a) + 2\pi] =$
 $= 2\sin(4\pi x + a) = h(x)$

$\forall x \in D_h, h\left(x + \frac{1}{2}\right) = h(x)$

Logo, h é uma função periódica de período fundamental

$P_0 = \frac{1}{2}$.

6.2. $h\left(\frac{1}{3}\right) = 0 \Leftrightarrow 2\sin\left(4\pi \times \frac{1}{3} + a\right) = 0 \Leftrightarrow$

$\Leftrightarrow \sin\left(\frac{4\pi}{3} + a\right) = 0 \Leftrightarrow \frac{4\pi}{3} + a = k\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow a = -\frac{4\pi}{3} + k\pi, k \in \mathbb{Z}$

$k = 0 \Rightarrow a = -\frac{4\pi}{3}$

$k = 1 \Rightarrow a = -\frac{\pi}{3}$

$k = 2 \Rightarrow a = \frac{2\pi}{3}$

Portanto, $a = \frac{2\pi}{3}$.

6.3. Se $x \in D_h$, então $-x \in D_h$, pois $D_h = \mathbb{R}$.

$\forall x \in \mathbb{R}, h(-x) = h(x) \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R}, 2\sin[4\pi(-x) + a] = 2\sin(4\pi x + a) \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R}, \sin(-4\pi x + a) = \sin(4\pi x + a) \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R}, -4\pi x + a = 4\pi x + a + 2k\pi \vee$

$\vee -4\pi x + a = \pi - (4\pi x + a) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R}, -4\pi x + a = \pi - 4\pi x - a + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow 2a = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow a = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

7. $D_f = \{x \in \mathbb{R} : \cos x \neq 0\} = \mathbb{R} \setminus \left\{x : x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

$D_g = \{x \in \mathbb{R} : \sin x \neq 1\} = \mathbb{R} \setminus \left\{x : x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}\right\}$

$D_f \cap D_g = D_f = D = \{x \in \mathbb{R} : \cos x \neq 0\}$

$f(x) = \frac{1 + \sin x}{\cos x} = \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$

$= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} = \frac{\cos^2 x}{\cos x(1 - \sin x)} = \frac{\cos x}{1 - \sin x} = g(x)$

Logo, em $D = \{x : \cos x \neq 0\}$ as funções f e g coincidem.

8. $f(x) = -1 + \sin x$ e $g(x) = 2\sin^2 x - 1$

$D_f = D_g = [0, 2\pi]$

8.1. $g(x) = 0 \Leftrightarrow 2\sin^2 x - 1 = 0 \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \sin^2 x = \frac{1}{2} \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \sin x = \pm \sqrt{\frac{1}{2}} \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \left(\sin x = \frac{\sqrt{2}}{2} \vee \sin x = -\frac{\sqrt{2}}{2} \right) \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \left(\sin x = \sin \frac{\pi}{4} \vee \sin x = \sin \left(-\frac{\pi}{4} \right) \right) \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} \vee x = \frac{3\pi}{4} \vee x = \frac{5\pi}{4} \vee x = \frac{7\pi}{4}$

$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

8.2. $f(x) = g(x) \Leftrightarrow$

$\Leftrightarrow -1 + \sin x = 2\sin^2 x - 1 \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow 2\sin^2 x - \sin x = 0 \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \sin x(2\sin x - 1) = 0 \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow \left(\sin x = 0 \vee \sin x = \frac{1}{2} \right) \wedge x \in [0, 2\pi] \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{\pi}{6} \vee x = \pi - \frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = \frac{\pi}{6} \vee x = \frac{5\pi}{6} \vee x = \pi \vee x = 2\pi$

$f(0) = -1 + \sin 0 = -1 = f(\pi) = f(2\pi)$

$f\left(\frac{\pi}{6}\right) = -1 + \sin \frac{\pi}{6} = -\frac{1}{2} = f\left(\frac{5\pi}{6}\right)$

Logo, $(0, -1)$, $\left(\frac{\pi}{6}, -\frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, -\frac{1}{2}\right)$, $(\pi, -1)$ e

$(2\pi, -1)$ são os pontos de interseção dos dois gráficos.

8.3. São os pontos de abscissa x tais que $|f(x) - g(x)| = 1$.

$|f(x) - g(x)| = 1 \Leftrightarrow$

$\Leftrightarrow |-1 + \sin x - (2\sin^2 x - 1)| = 1 \Leftrightarrow$

$\Leftrightarrow |2\sin^2 x - \sin x| = 1 \Leftrightarrow$

$\Leftrightarrow 2\sin^2 x - \sin x = 1 \vee 2\sin^2 x - \sin x = -1 \Leftrightarrow$

$\Leftrightarrow 2\sin^2 x - \sin x - 1 = 0 \vee 2\sin^2 x - \sin x + 1 = 0$

Fazendo $y = \sin x$, temos:

$2y^2 - y - 1 = 0 \vee 2y^2 - y + 1 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{1 \pm \sqrt{1+8}}{4} \vee y = \frac{1 \pm \sqrt{1-8}}{4} \Leftrightarrow$

$\Leftrightarrow y = -\frac{1}{2} \vee y = 1 \Leftrightarrow$

$\Leftrightarrow \sin x = -\frac{1}{2} \vee \sin x = 1$

No intervalo $[0, 2\pi]$, temos:

$x = \frac{\pi}{2} \vee x = \pi + \frac{\pi}{6} \vee x = 2\pi - \frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} \vee x = \frac{7\pi}{6} \vee x = \frac{11\pi}{6}$

$f\left(\frac{\pi}{2}\right) = -1 + \sin \frac{\pi}{2} = 0$; $g\left(\frac{\pi}{2}\right) = 2\sin^2 \frac{\pi}{2} - 1 = 1$

$f\left(\frac{7\pi}{6}\right) = -1 + \sin \frac{7\pi}{6} = -\frac{3}{2}$; $g\left(\frac{7\pi}{6}\right) = 2\sin^2 \frac{7\pi}{6} - 1 = -\frac{1}{2}$

$f\left(\frac{11\pi}{6}\right) = -1 + \sin \frac{11\pi}{6} = -\frac{3}{2}$

$g\left(\frac{11\pi}{6}\right) = 2\sin^2 \frac{11\pi}{6} - 1 = -\frac{1}{2}$

Portanto:

$A\left(\frac{\pi}{2}, 0\right)$ e $B\left(\frac{\pi}{2}, 1\right)$; $A\left(\frac{7\pi}{6}, -\frac{3}{2}\right)$ e $B\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$

ou $A\left(\frac{11\pi}{6}, -\frac{3}{2}\right)$ e $B\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$

9. $2\cos(2x) + 3\tan(2x) = 0 \wedge x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \Leftrightarrow$

$\Leftrightarrow 2\cos(2x) + 3\frac{\sin(2x)}{\cos(2x)} = 0 \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow 2\cos^2(2x) + 3\sin(2x) = 0 \wedge \cos(2x) \neq 0 \wedge$

$\wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow 2(1 - \sin^2(2x)) + 3\sin(2x) = 0 \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],$

dado que se $2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos(2x) \neq 0 \Leftrightarrow$

$\Leftrightarrow 2 - 2\sin^2(2x) + 3\sin(2x) = 0 \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow 2\sin^2(2x) - 3\sin(2x) - 2 = 0 \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow \sin(2x) = \frac{3 \pm \sqrt{9+16}}{4} \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

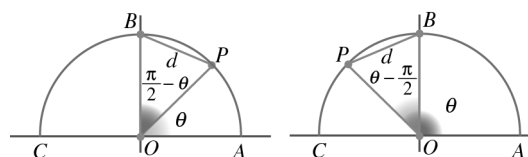
$\Leftrightarrow \left(\sin(2x) - \frac{1}{2} \vee \sin(2x) = 2 \right) \wedge 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow 2x = -\frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{12}$

$S = \left\{ -\frac{\pi}{12} \right\}$

10.1.



Se $0 < \theta \leq \frac{\pi}{2}$, $\widehat{POB} = \frac{\pi}{2} - \theta$.

Se $\frac{\pi}{2} < \theta < \pi$, $\widehat{POB} = \theta - \frac{\pi}{2}$.

$\cos\left(\theta - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

$\cos(\widehat{POB}) = \sin \theta$

Pelo Teorema de Carnot:

$$d^2 = \overline{OP}^2 + \overline{OB}^2 - 2 \times \overline{OP} \times \overline{OB} \times \cos(\widehat{POB})$$

$$d^2 = r^2 + r^2 - 2r \times r \times \sin \theta$$

$$d^2 = 2r^2 - 2r^2 \sin \theta$$

$$d^2 = 2r^2(1 - \sin \theta)$$

10.2. $d = r$

$$r^2 = 2r^2(1 - \sin \theta) \Leftrightarrow$$

$$\Leftrightarrow 1 - \sin \theta = \frac{r^2}{2r^2}$$

$$\Leftrightarrow 1 - \sin \theta = \frac{1}{2}$$

$$\Leftrightarrow \sin \theta = \frac{1}{2}$$

Como $\theta \in]0, \pi[$, $\sin \theta = \frac{1}{2} \Leftrightarrow$

$$\Leftrightarrow \theta = \frac{\pi}{6} \vee \theta = \pi - \frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow \theta = \frac{\pi}{6} \vee \theta = \frac{5\pi}{6}$$

Se $d = r$ o triângulo $[OPB]$ é equilátero pelo que $\widehat{POB} = \frac{\pi}{3}$.

Então, $\frac{\pi}{2} - \theta = \frac{\pi}{3}$ ou $\theta - \frac{\pi}{2} = \frac{\pi}{3}$. Logo, $\theta = \frac{\pi}{6}$ ou $\theta = \frac{5\pi}{6}$.

10.3. Seja y a ordenada de P .

$$\text{Então, } \frac{y}{r} = \sin \theta \vee \frac{y}{r} = \sin(\pi - \theta) \Leftrightarrow$$

$$\Leftrightarrow y = r \sin \theta \vee y = r \sin \theta \Leftrightarrow$$

$$\Leftrightarrow y = r \sin \theta$$

$$\text{Se } y = \frac{r}{9}, \text{ temos: } r \sin \theta = \frac{r}{9} \Leftrightarrow \sin \theta = \frac{1}{9}$$

$$\text{Se } \sin \theta = \frac{1}{9}: d^2 = 2r^2 \left(1 - \frac{1}{9}\right) \Leftrightarrow d^2 = \frac{16}{9}r^2 \Leftrightarrow d = \frac{4}{3}r$$

10.4. Base do triângulo $[OPB]$: $[OB]$

Altura h do triângulo $[OBP]$:

$$\text{Se } 0 < \theta \leq \frac{\pi}{2}, \frac{h}{r} = \cos \theta \Leftrightarrow h = r \cos \theta$$

$$\text{Se } \frac{\pi}{2} < \theta < \pi, \frac{h}{r} = \cos(\pi - \theta) \Leftrightarrow h = r(-\cos \theta)$$

Logo, $h = r|\cos \theta|$.

$$A_{[OBP]} = \frac{\overline{OB} \times h}{2} = \frac{r \times r|\cos \theta|}{2} = \frac{r^2}{2}|\cos \theta|$$

10.5. $r = 2$; $A_{[OBP]} = \frac{\sqrt{15}}{4}$

$$\frac{2^2}{2}|\cos \theta| = \frac{\sqrt{15}}{4} \Leftrightarrow 2|\cos \theta| = \frac{\sqrt{15}}{4} \Leftrightarrow |\cos \theta| = \frac{\sqrt{15}}{8}$$

$$\sin^2 \theta + \left(\frac{\sqrt{15}}{8}\right)^2 = 1 \Leftrightarrow \sin^2 \theta = 1 - \frac{15}{64}$$

$$\Leftrightarrow \sin^2 \theta = \frac{49}{64} \Leftrightarrow$$

$$\Leftrightarrow \sin \theta = \frac{7}{8}$$

$$d^2 = 2r^2(1 - \sin \theta)$$

$\sin \theta > 0$

$$d^2 = 2 \times 4 \left(1 - \frac{7}{8}\right) \Leftrightarrow d^2 = 8 - 7 \Leftrightarrow d = 1$$

Comprimento do arco $PB =$

$$= r \times \widehat{POB} = 2\widehat{POB}$$

$$\text{Se } 0 < \theta \leq \frac{\pi}{2}, \cos \theta = \frac{\sqrt{15}}{8} \Leftrightarrow \theta = \arccos \frac{\sqrt{15}}{8}$$

$$\widehat{POB} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \arccos \frac{\sqrt{15}}{8}$$

$$2\widehat{POB} = 2\left(\frac{\pi}{2} - \arccos \frac{\sqrt{15}}{8}\right) \approx 1,01$$

$$\text{Se } \frac{\pi}{2} < \theta \leq \pi, \cos \theta = -\frac{\sqrt{15}}{8} \Leftrightarrow \theta = \arccos \left(-\frac{\sqrt{15}}{8}\right)$$

$$\widehat{POB} = \theta - \frac{\pi}{2} = \arccos \left(-\frac{\sqrt{15}}{8}\right) - \frac{\pi}{2}$$

$$2\widehat{POB} = 2\left[\arccos \left(-\frac{\sqrt{15}}{8}\right) - \frac{\pi}{2}\right] \approx 1,01$$

Portanto, o comprimento do arco é aproximadamente 1,01.

Avaliação global

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1. $\widehat{CBA} = 90^\circ$

Como $\overline{AB} = \overline{BC}$, então:

$$\widehat{BCA} = \widehat{BAC} = \frac{180^\circ - 90^\circ}{2} = 45^\circ$$

$$\frac{\sin 45^\circ}{x} = \frac{\sin 110^\circ}{1} \Leftrightarrow x = \frac{\sin 45^\circ}{\sin 110^\circ}$$

$$\widehat{BDC} = 180^\circ - 110^\circ = 70^\circ$$

$$\frac{\sin 45^\circ}{x} = \frac{\sin 70^\circ}{1} \Leftrightarrow x = \frac{\sin 45^\circ}{\sin 70^\circ}$$

Resposta: (B)

2. $A(1, \tan \alpha)$; $\tan \alpha = \frac{3}{4}$; $\widehat{AOB} = \frac{\pi}{2}$

$$B\left(\cos\left(\frac{\pi}{2} + \alpha\right), \sin\left(\frac{\pi}{2} + \alpha\right)\right)$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + \left(\frac{3}{4}\right)^2 = \frac{1}{\cos^2 \alpha} \Leftrightarrow \frac{1}{\cos^2 \alpha} = \frac{25}{16} \Leftrightarrow \cos^2 \alpha = \frac{16}{25}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \frac{16}{25} = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{16}{25} \Leftrightarrow \sin^2 \alpha = \frac{9}{25}$$

$$\text{Como } \alpha \in 1.^\circ \text{ Q, } \sin \alpha = \frac{3}{5}, \text{ logo } -\sin \alpha = -\frac{3}{5}.$$

Resposta: (C)

3. $\cos^2 x = 1 \Leftrightarrow \cos x = -1 \vee \cos x = 1$

No intervalo $[0, 2\pi[$, a equação $\cos^2 x = 1$ tem duas soluções: $x = 0 \vee x = \pi$

Como a função cosseno é periódica de período fundamental 2π , em cada um dos 500 intervalos $[0, 2\pi[$,

$[2\pi, 4\pi[, \dots , [998\pi, 1000\pi[$ a equação tem duas soluções.

Dado que $\cos^2(1000\pi) = \cos^2 0 = 1$, a equação tem, no intervalo $[0, 1000\pi]$, $500 \times 2 + 1 = 1001$ soluções.

Resposta: (D)

4. $f(-x) = -f(x), \forall x \in \mathbb{R}$
 $f(-0) = -f(0) \Leftrightarrow f(0) + f(0) = 0 \Leftrightarrow$
 $\Leftrightarrow 2f(0) = 0 \Leftrightarrow f(0) = 0$

Resposta: (A)

5. α é solução da equação $\cos x = a$. Então, $\cos \alpha = a$.

A: $\cos(\pi + \alpha) = -a$?

$$\cos(\pi + \alpha) = -\cos \alpha = -a$$

Resposta: (A)

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6.1. $d = \overline{AB}$

$$d^2 = \overline{OA}^2 + \overline{OB}^2 - 2\overline{OA} \times \overline{OB} \times \cos(\widehat{BOA})$$

$$d^2 = 56,7^2 + 56,7^2 - 2 \times 56,7 \times 56,7 \times \cos(3,99^\circ)$$

$$d^2 \approx 15,5844$$

$$d \approx \sqrt{15,5844}$$

$$\overline{AB} \approx 3,95 \text{ m}$$

6.2. $\cos \alpha = \frac{\overline{OA}^2 + \overline{OB}^2 - \overline{AB}^2}{2 \times \overline{OA} \times \overline{OB}}$

$$\cos \alpha = \frac{56,7^2 + 56,7^2 - 5,44^2}{2 \times 56,7 \times 56,7}$$

$$\cos \alpha \approx 0,9954$$

$$\alpha \approx \cos^{-1}(0,9954) \approx 5,5^\circ$$

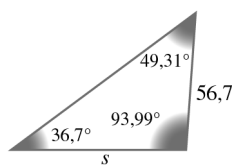
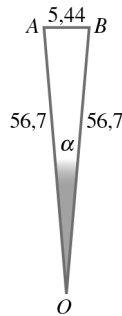
6.3. $90^\circ + 3,99^\circ = 93,99^\circ$

$$180^\circ - 93,99^\circ - 36,7^\circ = 49,31^\circ$$

$$\frac{\sin 49,31^\circ}{s} = \frac{\sin 36,7^\circ}{56,7}$$

$$s = \frac{56,7 \times \sin 49,31^\circ}{\sin 36,7^\circ}$$

$$s \approx 71,9 \text{ m}$$



7. $\overline{AC} = 2$
 $\widehat{BAC} = 90^\circ$
 $\overline{DC} = \overline{DB}$
 $\cos \alpha = \frac{\sqrt{5}}{3}$

7.1. Seja $\overline{CD} = x$. Então, $\overline{DB} = x$.

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$\left(\frac{\sqrt{5}}{3}\right)^2 + \sin^2 \alpha = 1 \Leftrightarrow \sin^2 \alpha = 1 - \frac{5}{9} \Leftrightarrow \sin^2 \alpha = \frac{4}{9}$$

Como α é um ângulo agudo, $\sin \alpha = \frac{2}{3}$.

$$\frac{\overline{AC}}{\overline{BC}} = \sin \alpha$$

$$\frac{2}{\overline{BC}} = \frac{2}{3} \Leftrightarrow 6 = 2\overline{BC} \Leftrightarrow \overline{BC} = 3$$

Pelo Teorema de Carnot:

$$\overline{DC}^2 = \overline{BC}^2 + \overline{BD}^2 - 2\overline{BC} \times \overline{BD} \times \cos \alpha$$

$$x^2 = 3^2 + x^2 - 2 \times 3 \times x \times \frac{\sqrt{5}}{3} \Leftrightarrow 2\sqrt{5}x = 9 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{9}{2\sqrt{5}} \Leftrightarrow x = \frac{9 \times \sqrt{5}}{2 \times \sqrt{5} \times \sqrt{5}} \Leftrightarrow x = \frac{9\sqrt{5}}{10}$$

$$\text{Logo, } \overline{CD} = \frac{9\sqrt{5}}{10}.$$

7.2. $\frac{\sin \theta}{\overline{BC}} = \frac{\sin \alpha}{x}$ (Lei dos senos)

$$\frac{\sin \theta}{3} = \frac{\frac{2}{9\sqrt{5}}}{\frac{10}{9\sqrt{5}}} \Leftrightarrow \sin \theta = 3 \times \frac{2 \times 10}{3 \times 9\sqrt{5}} \Leftrightarrow$$

$$\Leftrightarrow \sin \theta = \frac{20\sqrt{5}}{9 \times \sqrt{5} \times \sqrt{5}} \Leftrightarrow \sin \theta = \frac{4\sqrt{5}}{9}$$

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8. $26^\circ 42' = \left(26 + \frac{42}{60}\right)^\circ = 26,7^\circ$

$$13^\circ 18' = \left(13 + \frac{18}{60}\right)^\circ = 13,3^\circ$$

$$\theta = 26,7^\circ - 13,3^\circ = 13,4^\circ$$

$$\frac{\overline{SB}}{\overline{OB}} = \tan \theta \Leftrightarrow \overline{SB} = \overline{OB} \tan \theta$$

$$\overline{SB} = 6370 \times \tan(13,4^\circ) \Leftrightarrow$$

$$\Leftrightarrow \overline{SB} \approx 1518 \text{ km}$$

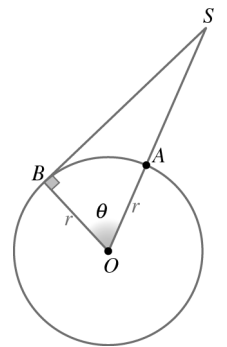
$$\frac{\overline{OB}}{\overline{OS}} = \cos \theta$$

$$\frac{6370}{6370 + \overline{AS}} = \cos 13,4^\circ$$

$$6370 = 6370 \times \cos 13,4^\circ + \overline{AS} \times \cos 13,4^\circ$$

$$\overline{AS} = \frac{6370 - 6370 \times \cos 13,4^\circ}{\cos 13,4^\circ}$$

$$\overline{AS} \approx 178 \text{ km}$$



9. $r = 6370 \text{ km}$
 $h = 20\,200 \text{ km}$

$$\frac{r}{h+r} = \cos \theta$$

$$\frac{6370}{20\,200 + 6370} = \cos \theta$$

$$\cos \theta = \frac{6370}{26\,570}$$

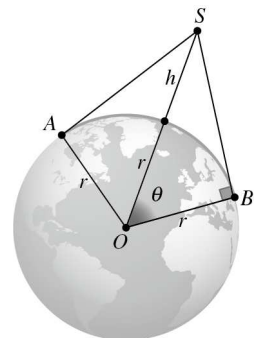
$$2\theta = 2 \arccos\left(\frac{637}{2657}\right)$$

Comprimento do arco AB =

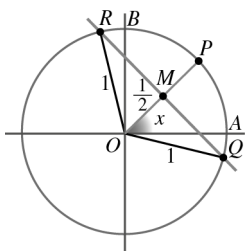
$$= 2\theta \times r \quad (\theta \text{ em radianos})$$

$$= 2 \times \arccos\left(\frac{637}{2657}\right) \times 6370$$

$$\approx 16\,927 \text{ km}$$



10.



$$10.1. \overline{OA} = \overline{OB} = \overline{OR} = \overline{OQ} = 1$$

$$\overline{OM} = \frac{1}{2}$$

Os triângulos $[OQM]$ e $[OMR]$ são iguais e retângulos em M .
 $\cos x =$ abscissa de P

$$\cos x = \frac{\sqrt{2}}{2}. \text{ Logo, } \widehat{AOP} = x = \frac{\pi}{4}.$$

$$\frac{\overline{OM}}{\overline{OQ}} = \cos(\widehat{QOM})$$

$$\frac{1}{2} = \cos(\widehat{QOM})$$

$$\text{Logo, } \widehat{QOM} = \frac{\pi}{3}.$$

$$\widehat{QOA} = \widehat{QOM} - \widehat{AOM} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Uma amplitude do ângulo orientado de lado origem $\widehat{OA}$ e
lado extremidade $\widehat{OQ}$ é, por exemplo, $-\frac{\pi}{12}$ rad.

$$10.2. \widehat{AOR} = \frac{\pi}{2} + \frac{\pi}{12} = \frac{7\pi}{12}$$

Uma amplitude do ângulo orientado $\widehat{AOR}$ é, por exemplo,
 $\frac{7\pi}{12}$.

$$11.1. \tan^2 \alpha - \sin^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} - \sin^2 \alpha =$$

$$= \frac{\sin^2 \alpha - \sin^2 \alpha \cos^2 \alpha}{\cos^2 \alpha} =$$

$$= \frac{\sin^2 \alpha (1 - \cos^2 \alpha)}{\cos^2 \alpha} =$$

$$= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \sin^2 \alpha =$$

$$= \tan^2 \alpha \times \sin^2 \alpha$$

$$11.2. \frac{\sin \alpha}{1 - \cos \alpha} - \frac{1}{\tan \alpha} = \frac{\sin \alpha}{1 - \cos \alpha} - \frac{\cos \alpha}{\sin \alpha} =$$

$$= \frac{\sin^2 \alpha - \cos \alpha (1 - \cos \alpha)}{(1 - \cos \alpha) \sin \alpha} =$$

$$= \frac{\sin^2 \alpha - \cos \alpha + \cos^2 \alpha}{(1 - \cos \alpha) \sin \alpha} =$$

$$= \frac{(\sin^2 \alpha + \cos^2 \alpha) - \cos \alpha}{(1 - \cos \alpha) \sin \alpha} =$$

$$= \frac{1 - \cos \alpha}{(1 - \cos \alpha) \sin \alpha} =$$

$$= \frac{1}{\sin \alpha}$$

$$12.1. 1 - \frac{\cos x}{\sin x} = \frac{1}{\sin^2 x} \Leftrightarrow \frac{\sin^2 x - \sin x \cos x - 1}{\sin^2 x} = 0 \Leftrightarrow$$

$$\Leftrightarrow -(1 - \sin^2 x) - \sin x \cos x = 0 \wedge \sin^2 x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow -\cos^2 x - \sin x \cos x = 0 \wedge \sin x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x (-\cos x - \sin x) = 0 \wedge \sin x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow (\cos x = 0 \vee \cos x + \sin x = 0) \wedge \sin x \neq 0 \Leftrightarrow$$

$$\Leftrightarrow \cos x = 0 \vee \cos x = -\sin x \Leftrightarrow$$

$$\Leftrightarrow \cos x = 0 \vee \cos x = \sin(-x) \Leftrightarrow$$

$$\Leftrightarrow \cos x = 0 \vee \cos x = \cos\left(\frac{\pi}{2} - (-x)\right) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi \vee x = \frac{\pi}{2} + x + 2k\pi \vee$$

$$\vee x = -\frac{\pi}{2} - x + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi \vee \text{condição impossível} \vee$$

$$\vee 2x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\pi \vee x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$12.2. \sin x + \cos\left(x - \frac{\pi}{4}\right) = 0 \wedge x \in]-\pi, \pi[$$

$$\sin x + \cos\left(x - \frac{\pi}{4}\right) = 0 \Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = -\sin x \Leftrightarrow$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} + x\right) \Leftrightarrow$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} + x\right) \Leftrightarrow$$

$$\Leftrightarrow x - \frac{\pi}{4} = \frac{\pi}{2} + x + 2k\pi \vee x - \frac{\pi}{4} = -\frac{\pi}{2} - x + 2k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow \text{condição impossível} \vee 2x = -\frac{\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{8} + k\pi, k \in \mathbb{Z}$$

$$k = -1 \Rightarrow x = -\frac{\pi}{8} - \pi \times \quad k = 0 \Rightarrow x = -\frac{\pi}{8} \checkmark$$

$$k = 1 \Rightarrow x = -\frac{\pi}{8} + \pi = \frac{7\pi}{8} \checkmark \quad k = 2 \Rightarrow x = -\frac{\pi}{8} + 2\pi \times$$

$$S = \left\{ -\frac{\pi}{8}, \frac{7\pi}{8} \right\}$$

$$13. \text{ Se } x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[, \cos x \neq 0.$$

$$\sin x + 2\cos x = 0 \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \frac{\sin x}{\cos x} + \frac{2\cos x}{\cos x} = \frac{0}{\cos x} \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \tan x + 2 = 0 \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \tan x = -2 \wedge x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow x = \arctan(-2) \Rightarrow$$

$$\Rightarrow x \approx -1,11 \text{ rad}$$

$$14.1. \overline{AQ} = \overline{OQ} - \overline{OA} = \overline{OQ} - r$$

$$\frac{r}{\overline{OQ}} = \cos \theta \Leftrightarrow \overline{OQ} = \frac{r}{\cos \theta}$$

$$\overline{AQ} = \frac{r}{\cos \theta} - r = \frac{r - r \cos \theta}{\cos \theta}$$

$$d = \frac{r(1 - \cos \theta)}{\cos \theta}$$

$$14.2. \text{ Se } d = r :$$

$$r = \frac{r(1 - \cos \theta)}{\cos \theta} \Leftrightarrow 1 = \frac{1 - \cos \theta}{\cos \theta} \Leftrightarrow$$

$$\Leftrightarrow \cos \theta = 1 - \cos \theta \Leftrightarrow \text{pois } \cos \theta \neq 0$$

$$\Leftrightarrow 2 \cos \theta = 1 \Leftrightarrow \cos \theta = \frac{1}{2}$$

$$\text{Como } \theta \in \left] 0, \frac{\pi}{2} \right[, \theta = \frac{\pi}{3}.$$

$$14.3. r = 1 \text{ e } d = 2$$

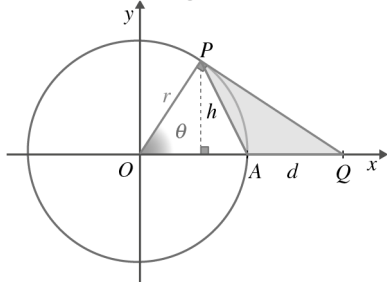
$$2 = \frac{1(1 - \cos \theta)}{\cos \theta} \Leftrightarrow 2 \cos \theta = 1 - \cos \theta \Leftrightarrow \cos \theta = \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^2 + \sin^2 \theta = 1 \Leftrightarrow \sin^2 \theta = 1 - \frac{1}{9} \Leftrightarrow \sin^2 \theta = \frac{8}{9}$$

$$\text{Como } \theta \in \left] 0, \frac{\pi}{2} \right[, \sin \theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}.$$

$$\text{Logo, } P\left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right).$$

$$14.4. \text{ a) Seja } h \text{ a altura do triângulo } [AQP], \text{ relativa ao vértice } P$$



$$\frac{h}{r} = \sin \theta \Leftrightarrow h = r \sin \theta$$

$$A_{[AQP]} = \frac{d \times h}{2} = \frac{1}{2} \times d \times h =$$

$$= \frac{1}{2} \times \frac{r(1 - \cos \theta)}{\cos \theta} \times r \sin \theta =$$

$$= \frac{1}{2} \times r^2 \times (1 - \cos \theta) \times \frac{\sin \theta}{\cos \theta}$$

$$A = \frac{r^2}{2} (1 - \cos \theta) \tan \theta$$

$$\text{b) } \tan \alpha + \frac{1}{\tan \alpha} = 2 \wedge \alpha \in \left] 0, \frac{\pi}{2} \right[$$

Como $\tan \alpha \neq 0$, vem:

$$\tan^2 \alpha + 1 = 2 \tan \alpha \wedge \alpha \in \left] 0, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \tan^2 \alpha - 2 \tan \alpha + 1 = 0 \wedge \alpha \in \left] 0, \frac{\pi}{2} \right[\Leftrightarrow$$

$$\Leftrightarrow \tan \alpha = \frac{2 \pm \sqrt{4 - 4}}{2} \wedge \alpha \in \left] 0, \frac{\pi}{2} \right[\Leftrightarrow$$

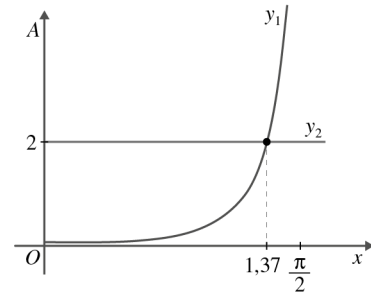
$$\Leftrightarrow \tan \alpha = 1 \wedge \alpha \in \left] 0, \frac{\pi}{2} \right[\Leftrightarrow \alpha = \frac{\pi}{4}$$

$$A\left(\frac{\pi}{4}\right) = \frac{2^2}{2} \times \left(1 - \cos \frac{\pi}{4}\right) \times \tan \frac{\pi}{4} =$$

$$= 2 \times \left(1 - \frac{\sqrt{2}}{2}\right) \times 1 =$$

$$= 2 - \sqrt{2}$$

- c) Fazendo, na calculadora gráfica, $y_1 = \frac{1}{2}(1 - \cos x) \tan x$ e $y_2 = 2$, determinou-se a abscissa do ponto de interseção dos respectivos gráficos, no intervalo $\left] 0, \frac{\pi}{2} \right[$:



Assim, $\theta \approx 1,37$.

15. Se P é o período de f , então:

$$(1) \bullet \text{ Se } x \in D_f, x + P \in D_f$$

$$(2) \bullet f(x + P) = f(x), \forall x \in D_f$$

Por (1):

$$\text{Se } x \in D_f, x + P \in D_f.$$

Logo:

$$\text{Se } (x + P) \in D_f, \text{ então } (x + P) + P \in D_f.$$

$$\text{Se } x \in D_f, \text{ então } x + 2P \in D_f.$$

Por (2):

$$f(x + 2P) = f((x + P) + P)$$

$$= f(x + P)$$

$$= f(x), \forall x \in D_f$$

Logo, se P é período de uma função f , então $2P$ também é período da função f .